Solving Word Problems Using Schemas

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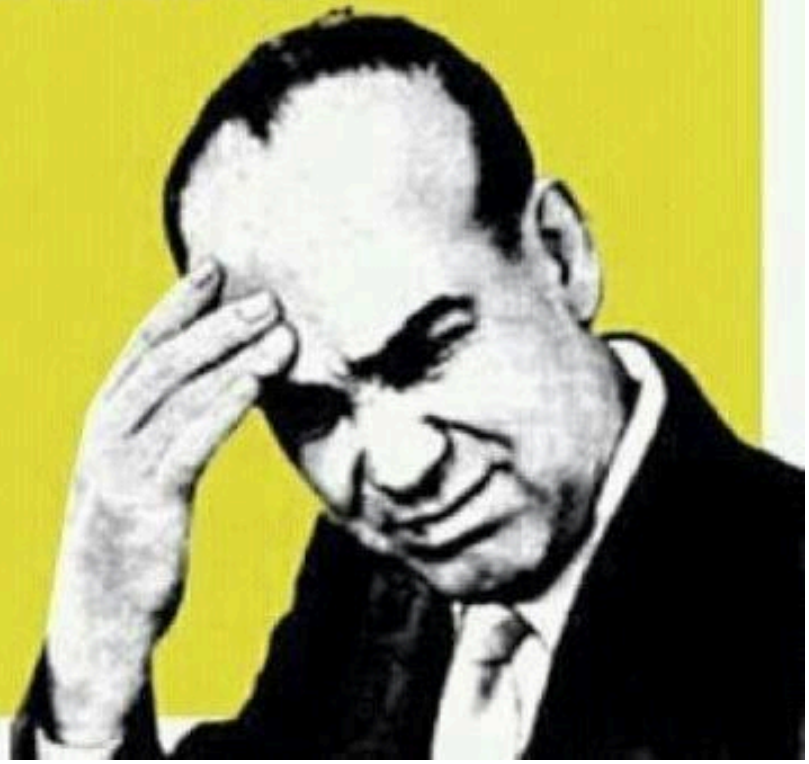
Everytime I see a math word problem it looks like this:
If I have 10 ice cubes and you have 11 apples.
How many pancakes will fit on the roof?
Answer:
Purple because aliens
don't wear hats.



arrg! ecards

Problem Solving

- Students have difficulty with:

Mrs. Stephens drove through a total of 36 intersections on her way home from work last week. At 4 of every 16 intersections, Mrs. Stephens had to stop for a red light before she could drive through. At how many intersections did Mrs. Stephens have to stop for a red light?

A 3

B 9

C 24

D 12

Problem Solving

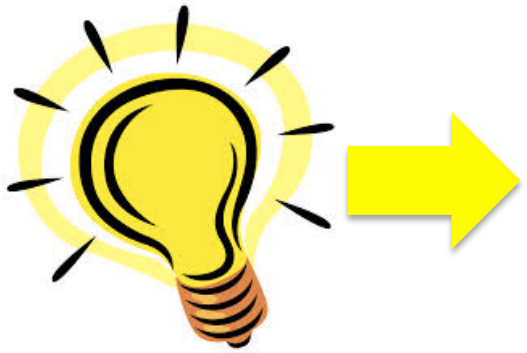
- Students have difficulty with:
  - Reading problems
  - Identifying relevant information (and irrelevant information)
  - Identifying appropriates operations
  - Performing computations
  - Translating words into number equations
  - Understanding vocabulary
  - Using high-level reasoning

Steps to Solving Problems

- According to Polya (1945)
- 1. Understand the problem
  - What information is provided?
  - What is being asked?
- 2. Devise a plan
  - Have you done a similar problem?
  - Can you paraphrase? Make a table? Make a simpler problem? Write a number sentence? Word backward?
- 3. Carry out the plan
  - Can you guess and check?
  - Can you use other strategies?
- 4. Look back and check
  - Can you check?

For every word problem

- Regardless of problem type, students need an "attack" strategy for working through the problem
- This strategy should work for any problem type (i.e., schema)



RIDGES

Read the problem.
I know statement.
Draw a picture.
Goal statement.
Equation development.
Solve the equation.

STAR

Search the word problem.
Translate the problem.
Answer the problem.
Review the solution.

SQRQCQ

Survey
Question
Read
Question
Compute
Question

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

DRAW

Discover the sign.
Read the problem.
Answer, or draw and check.
Write the answer.

SIGNS

Survey questions
Identify key words
Graphically draw problem
Note operations
Solve and check

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.

Read
Paraphrase
Hypothesize
Estimate
Compute
Check

General Strategies

- Draw a picture, act it out, use a model
  - Can use concrete objects or students themselves
- Look for a pattern
- Guess and check
  - Good, educated guesses
- Make a table or chart
  - Helps arrange information
- Try a simpler problem
- Make an organized list
- Write an equation

Key Words


combined
addition
in all
total
sum
together
plus
both
perimeter
add
more than
triple
twice
factor
product
multiply
each
per
in all
multiple
area
times
double
key words
average
subtract
decrease
fewer
remain
take away
minus
less than
how many more...
division
split
quotient
equal groups
divide
half
shared equally
each
distribute
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Key Words Used in Math Word Problems
Addition Words
+ add
+ all together or altogether
+ and
+ both
+ combined
+ how many in all
+ how much
+ in all
+ increased by
+ plus
+ sum
+ together
+ total
Subtraction Words
- change
- decreased by
- difference
- fewer or fewer than
- how many are left (or have left)
- how many did not have
- how many (or much) more
- how much longer (shorter, taller, heavier, etc.)
- less or less than
- lost
- minus
- need to
- reduce
- remain
- subtract
- take away
Multiplication Words
× by (dimension)
× double
× each group
× every
× factor of
× increased by
× multiplied by
× of
× product
× times
× triple
Division Words
÷ as much
÷ cut up
÷ each group has
÷ equal sharing
÷ half (or other fractions)
÷ how many in each
÷ parts
÷ per
÷ percent
÷ quotient of
÷ ratio of
÷ separated
÷ share something equally

Key Words

Addition
Sum
Plus
And
Total
Increase
More
Raise
Combined
In all
Altogether
Extra

- Kasey made $42, and Mandi made $37. How much money did they make altogether?
- Kasey and Mandi made $79 altogether. If Kasey made $42, how much money did Mandi make?
- Kasey mowed 12 lawns on Monday. Then, she mowed 10 more on Tuesday. How many lawns has Kasey mowed?
- Kasey mowed 22 lawns and Mandi mowed 7 lawns. How many more lawns did Kasey mow than Mandi?

Key Words

Subtraction
More than
Less than
Decrease
Difference
Reduce
Change
Left
Remain
Dropped
Lost
Nearer

- Becky has $70 more than Perla. If Becky has $120, how much money does Perla have?
- Becky has $70 more than Perla. If Perla has $50, how much money does Becky have?
- Becky had 9 dinosaurs and then she lost 4 of them. How many dinosaurs does Becky have now?
- Becky had some dinosaurs and then she lost 4 of them. Now Becky has 5 dinosaurs. How many dinosaurs did she start with?

Key Words

Multiplication
Product
Of
Multiplied
Times
As much
Lost
By
Twice
Multiplication
Product
Of

- Matt made 4 times as many cookies as Courtney. Courtney made 10 cookies. How many cookies did Matt make?
- Matt had to go to the store 4 times to buy ingredients for a cake. He used 3 sticks of butter for the cake and 2 sticks of butter for icing. How many sticks of butter did Matt use?

Key Words

Division
Divide
Evenly
Cut
Split
Each
Every
Out of
Shared
Average
Ratio
Quotient

- Rachel wants to share 36 brownies with 6 friends. How many cookies will each friend receive?
- Rachel shared brownies with 6 friends. Each friend ate 6 brownies. How many brownies did Rachel have to start with?

Key Words

Addition	Subtraction	Multiplication	Division
Sum			
Plus			
And			
Total			
Increase			
More			
Raise			
Combin			
In all			
Altogeth			
Extra	Nearer		Quotient

Instruction

- When first learning about word problems:
  - Focus on problem types **schema**
    - Have strategy for working through problems
    - Practice identifying problems by schema/type
  - Manipulatives help with conceptual understanding
  - Continue fluency practice

ADDITIVE SCHEMAS

Total/Combine/ Part-Part-Whole

Situation is static.

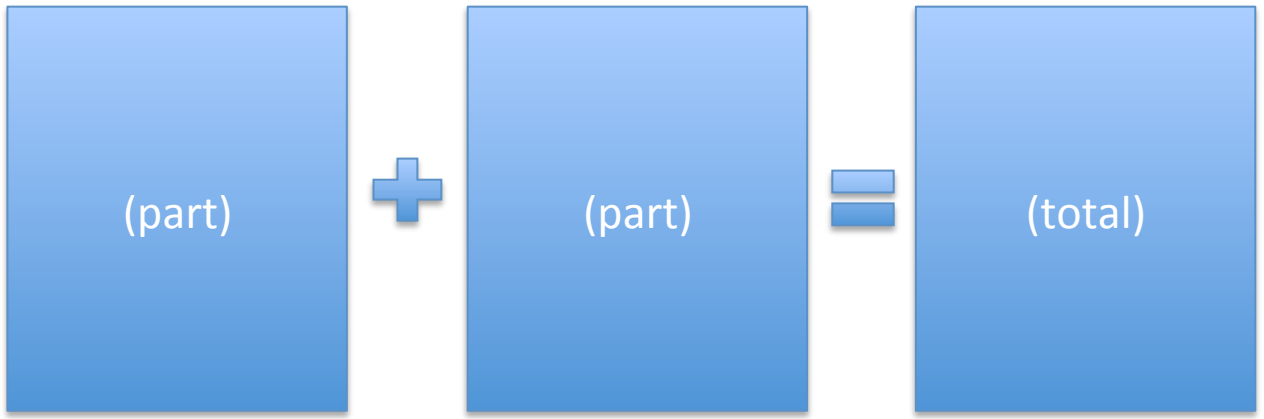
- **Parts** put together into a **total**

  P1 + P2 = T

- Examples
  - Emily saw **4** cardinals and **5** bluejays. How many birds did Emily see?
    - **4 + 5 = X**
  - Emily saw **9** birds. If **4** of the birds were cardinals, how many were bluejays?
    - **4 + X = 9**
  - Emily saw **9** birds. **5** of the birds were bluejays, how many were cardinals?
    - **5 + X = 9**

Total/Combine/ Part-Part-Whole

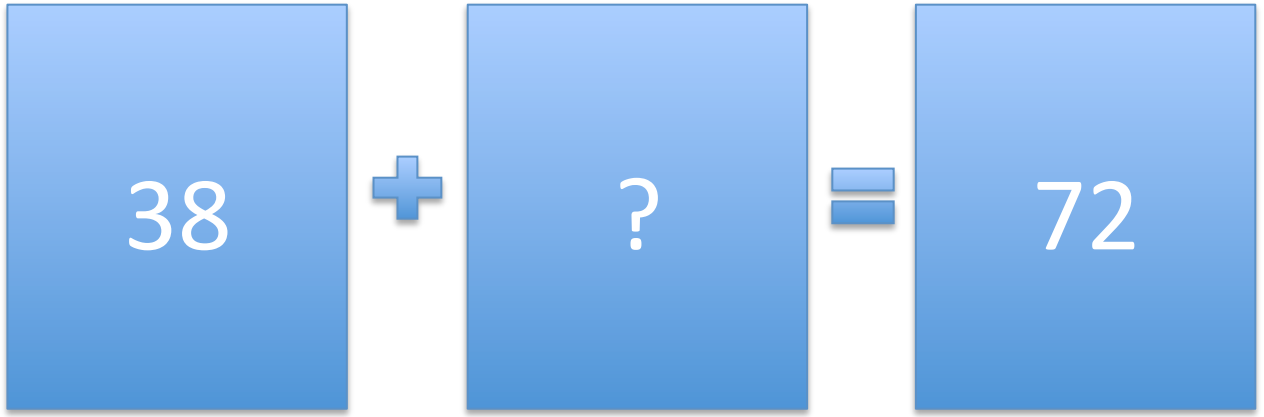
Situation is static.



Griffin & Jitendra (2009)

Total/Combine/ Part-Part-Whole

Situation is static.



Melissa has 72 roses and tulips in her garden. If 38 of the flowers are roses, how many are tulips?

$38 + ? = 72$

$$\begin{array}{r} 72 \\ -\ ? \\ \hline \end{array}$$

34 tulips

Difference/Compare

Situation is static.

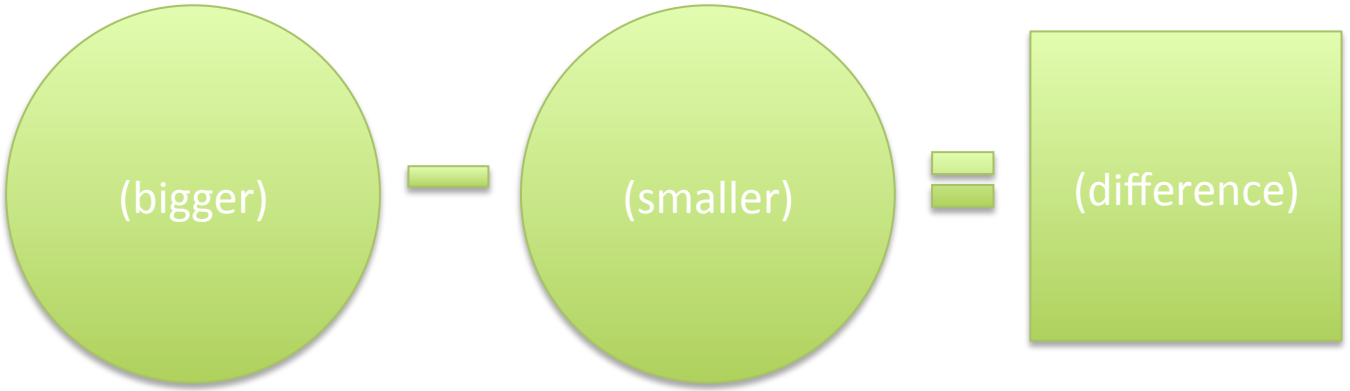
- Bigger and smaller amounts compared for a difference

  B – s = D

- Examples
  - Shinead has **9** apples. Amanda has **4** apples. How many more apples does Shinead have? (How many fewer?)
    - **9 – 4 = X**
  - Shinead has **5** more apples than Amanda. If Amanda has **4** apples, how many does Shinead have?
    - **X – 4 = 5**
  - Amanda has **5** fewer apples than Shinead. Shinead has **9** apples. How many apples does Amanda have?
    - **9 – X = 5**

Difference/Compare

Situation is static.



Griffin & Jitendra (2009)

Difference/Compare

Situation is static.

Paulo is 3 feet taller than his baby brother. His baby brother is 2 feet tall. How tall is Paulo?

? – 2 = 3

3
+ 2
5 feet tall

Griffin & Jitendra (2009)

Change/Join

Situation occurs over time.

- An amount that **increases** or decreases

  ST + **C** = **E**

- Examples
  - Shannah had $**4**. Then she got $**3** for cleaning her room. How much money does Shannah have now?
    - **4** + **3** = X
  - Shannah has $**4**. Then she earned money for cleaning her room. Now Shannah has $**7**. How much money did she earn?
    - **4** + **X** = **7**
  - Shannah had some money. Then she made $**3** for cleaning her room. Now she has $**7**. How much money did Shannah start with?
    - **X** + **3** = **7**

Change/Separate

Situation occurs over time.

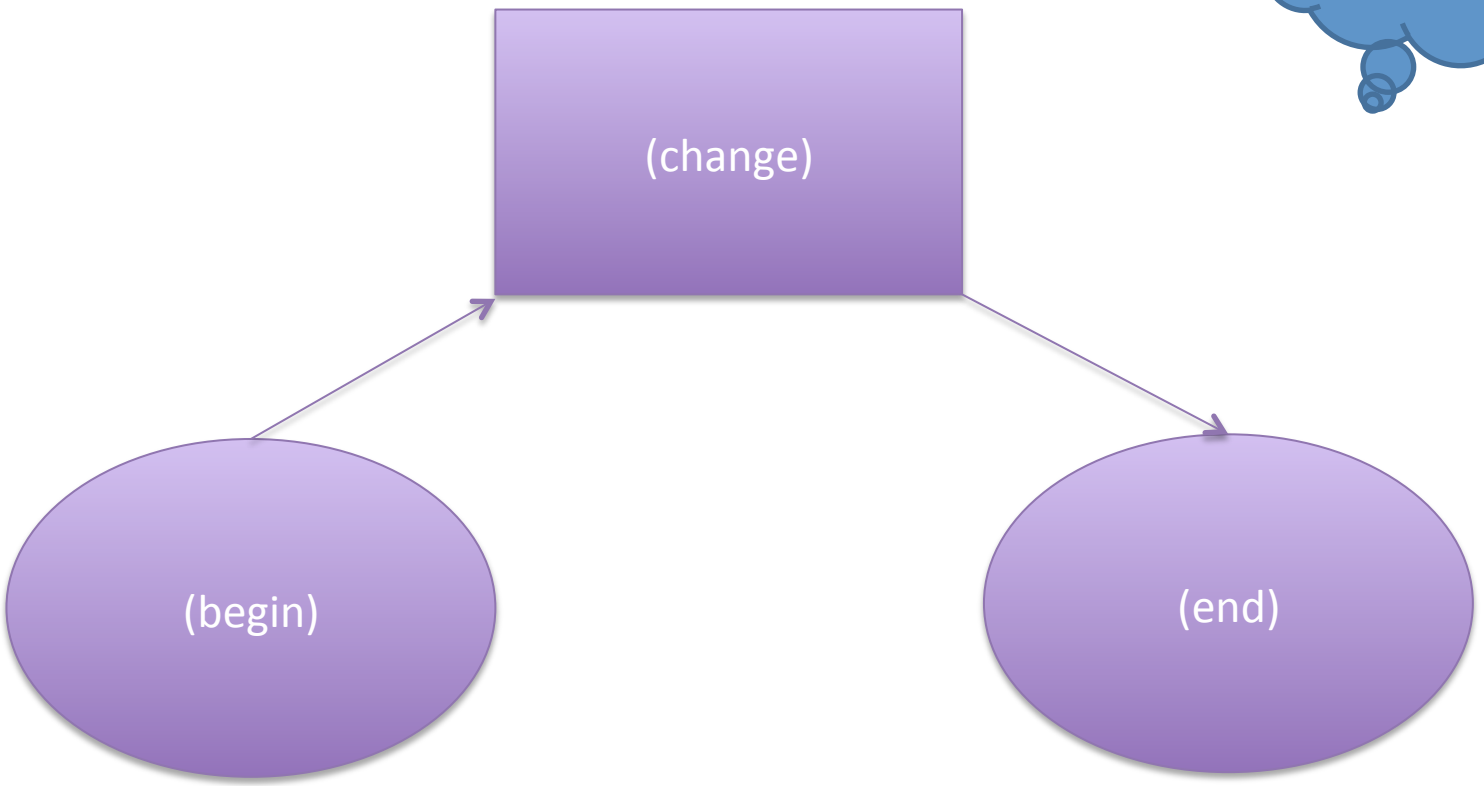
- An amount that increases or **decreases**

  ST – C = E

- Examples
  - Micaela had $**9**. Then she spent $**2** on candy. How much money does Micaela have now?
    - **9 – 2 = X**
  - Micaela had $**9**. She spent some money on candy. Now Micaela has $**2**. How much money did Micaela spend on candy?
    - **9 – X = 2**
  - Micaela had some money. Then she spent $**2** on candy. Now she has $**7**. How much money did Micaela start with?
    - **X – 2 = 7**

Change

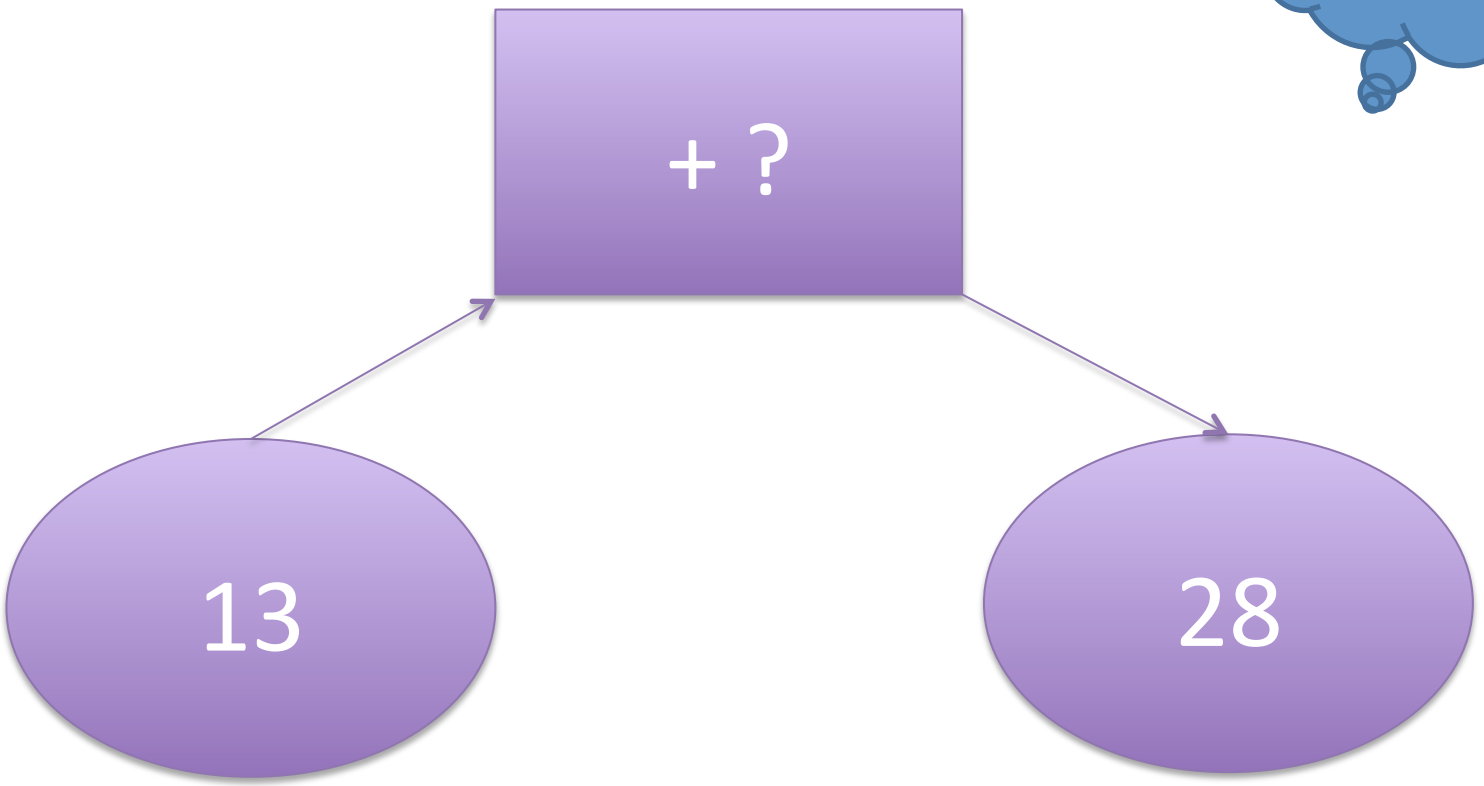
Situation occurs over time.



Griffin & Jitendra (2009)

Change

Situation occurs over time.



A bus had 13 passengers. At the next stop, more passengers got on the bus. Now, there are 28 passengers. How many passengers got on the bus?

$13 + ? = 28$

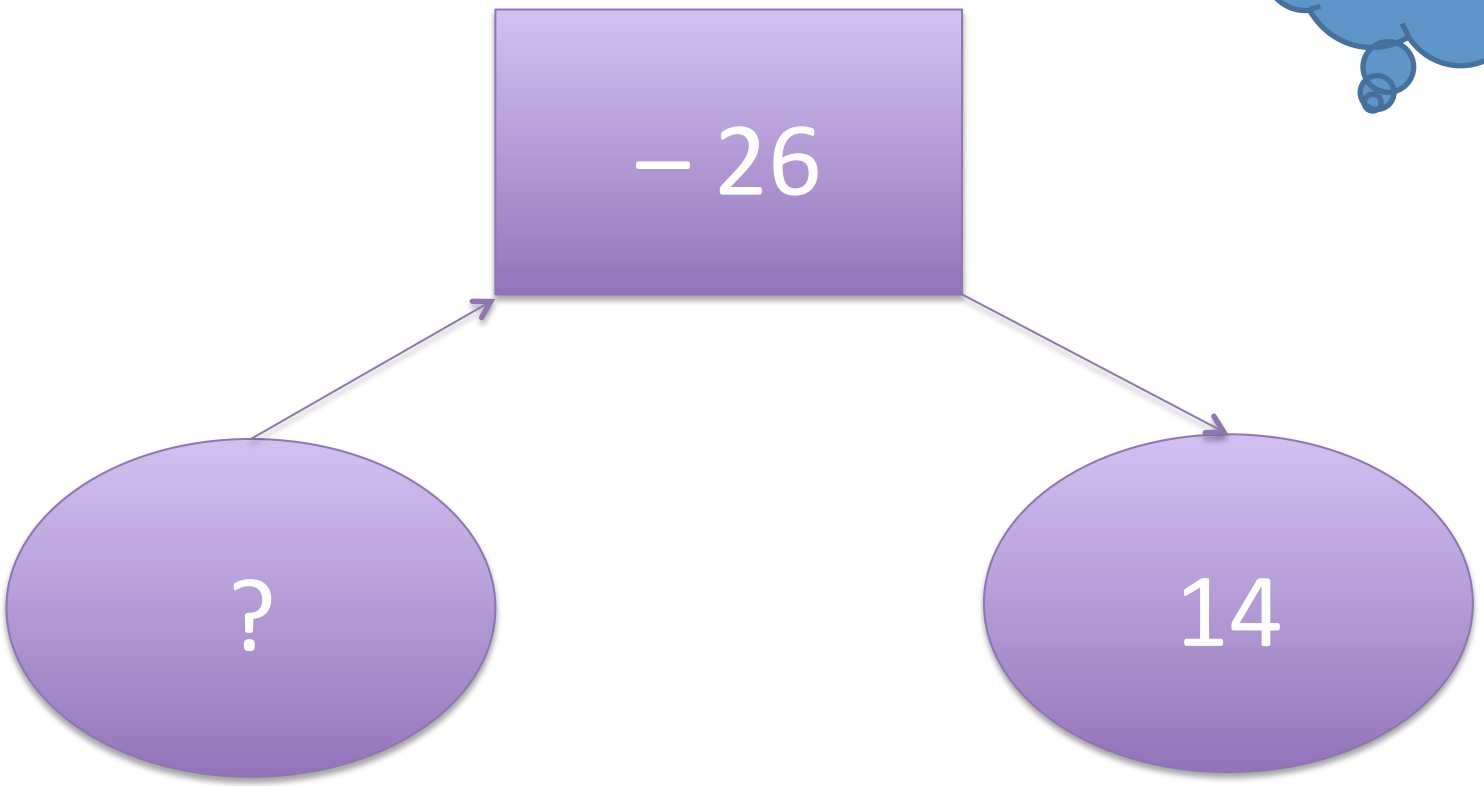
$$\begin{array}{r} 28 \\ -\ 13 \\ \hline \end{array}$$

15 passengers

Griffin & Jitendra (2009)

Change

Situation occurs over time.



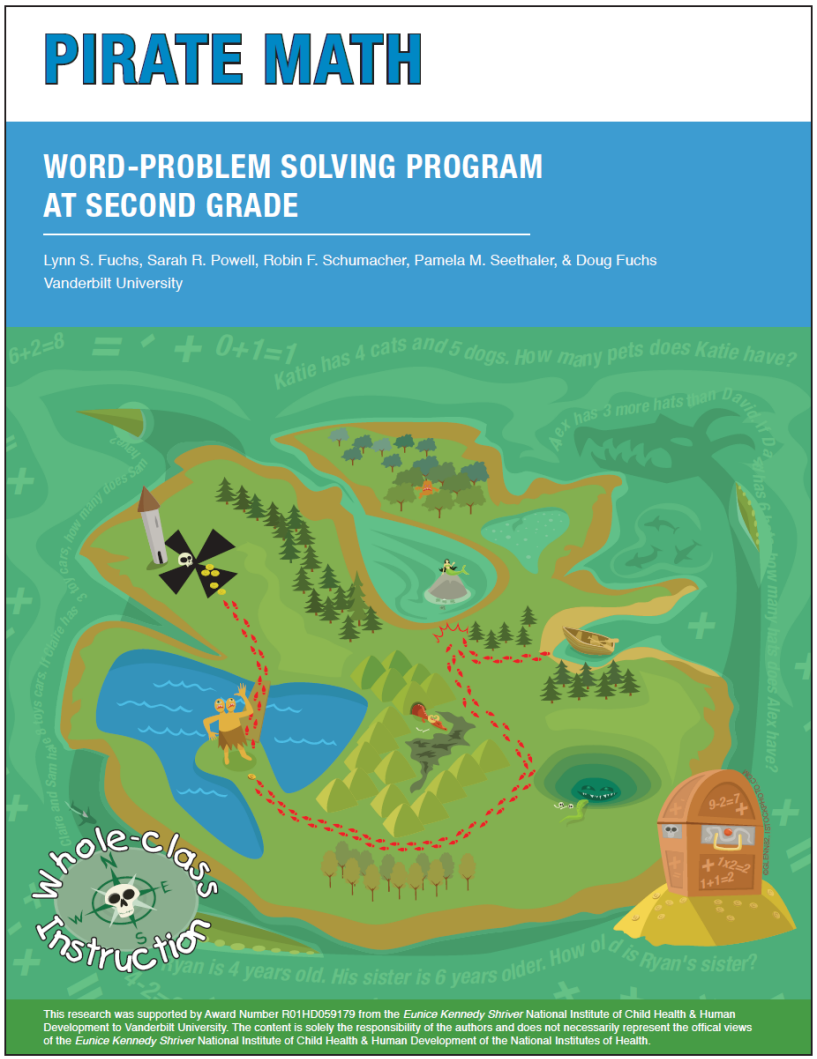
Michael had some money. Then, we spent $26 on groceries. Now, he has $14. How much money did Michael have to start with?

? – 26 = 14

26
+ 14
$40

Griffin & Jitendra (2009)

EVIDENCE-BASED PRACTICE



Fuchs, Powell, et al. (2014)
Fuchs, Powell, et al. (2010)

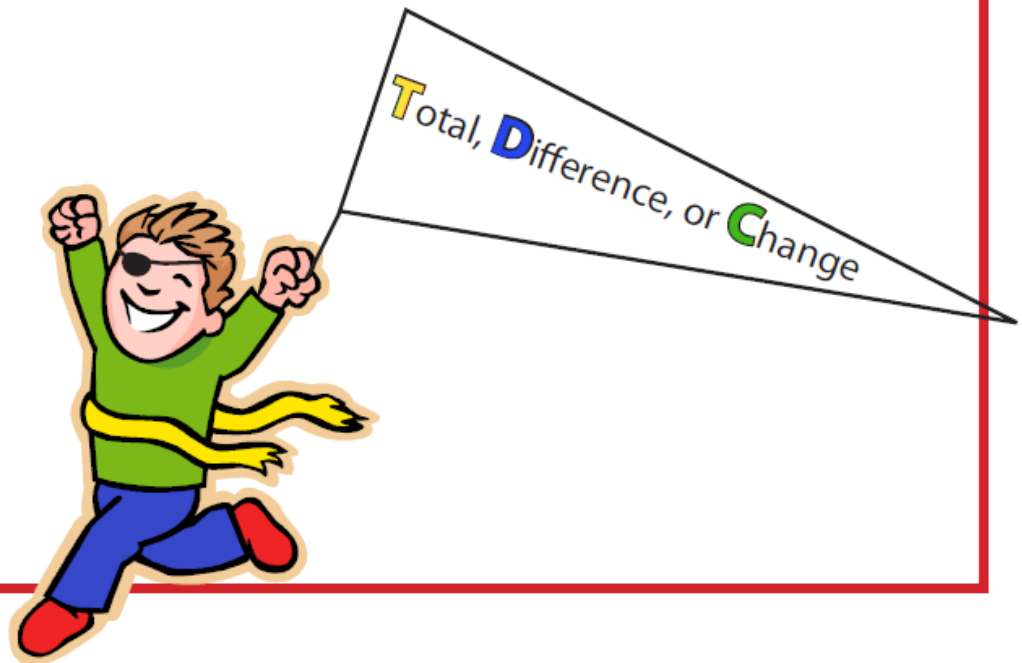
- RUN! is used for every problem, regardless of problem type

RUN!

1. ***Read*** **the problem.**

2. ***Underline*** **the labels.**

3. ***Name*** **the problem type.**



- Students use equations to represent the word problem information
- X marks the missing information
- Students learn strategies for "finding X"

Find X

Is **X** at the end?

Solve it!

$2 + 3 = X$

$5 - 2 = X$

Is it **X –** ?

Add

$X - 2 = 3$

Otherwise:

Subtract

$X + 2 = 5$

$3 + X = 5$

$5 - X = 3$

RUN!

1. *Read* the problem.
2. *Underline* the labels.
3. *Name* the problem type.

TOTAL

1. Write P1 + P2 = T.
2. What's T?
3. What's P1 and P2?
4. Write the signs.
5. Find X!

P1 + P2 = T

T Mr. Jones has 8 pet crickets and 3 pet fish in his classroom. He also has 6 plants. How many pets does Mr. Jones have in his classroom?

P1 + P2 = T

8 + 3 = X

8 + 3 = 11

X = 11 pets

TOTAL

1. Write P1 + P2 = T.
2. What's T?
3. What's P1 and P2?
4. Write the signs.
5. Find X!

P1 + P2 = T

Ella and Susan planted ~~12~~ ✓ flower seeds. Ella planted ~~3~~ ✓ flower seeds ~~and 2 trees.~~ How many flower seeds did Susan plant?

P1 + P2 = T

3 + X = 12

3 + 9 = 12

X = 9 flower seeds

DIFFERENCE

1. Write B – s = D.

2. What's the compare sentence?
   - Does it give D?

3. What's B and s?

4. Write the signs.

5. Find X!

B – s = D

Answer Sheet

D

B s

Farmer Hank has ~~a~~ more cows than horses. He has horses. ~~He also has 9 chickens.~~ How many cows does he have?

B – s = D

X – 4 = 6

10 – 4 = 6

X = 10 cows

CHANGE

1. Write ST + C = E or ST - C = E.
2. What's ST?
3. What's C?
4. What's E?
5. Write the signs.
6. Find X!

ST + C = E

ST – C = E

+C

In the morning, it rained 2 inches. Then in the afternoon it rained some more. Over the whole day, it rained 5 inches. How many inches did it rain in the afternoon?

ST + C = E

2 + X = 5

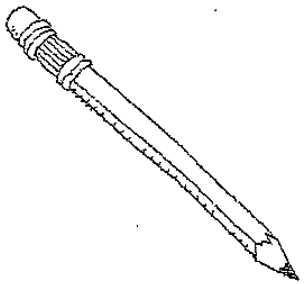
2 + 3 = 5

X = 3 inches

53. Antonio has 29 cookies. Thirteen of the cookies are chocolate chip. The rest are oatmeal. How many cookies are oatmeal?

Ramón has a total of 815 sheep in two fields. He has 348 sheep in one of the fields. How many sheep does Ramón have in the other field?

Max had 7 pencils in his desk.
He gave some of the pencils away.
He had 4 pencils left in his desk.
How many pencils did Max give away?

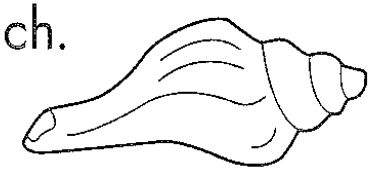


Mrs. Lanier saved $617 in January. In February she spent $249 of the money she had saved. She saved $291 more in March. Which number sentence can be used to find the amount of money Mrs. Lanier had at the end of March?

There were 17 apples in a basket and 63 apples in a barrel. How many fewer apples were in the basket than were in the barrel?

168. The little dog has 13 spots on his back. The big dog has 5 more spots than the little one. How many spots do they have altogether?

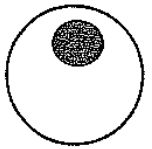
Max, Rosa, and Kim went to the beach.
Kim found 3 shells.
Rosa found 2 shells.
Max found 7 shells.
How many shells did they find in all?

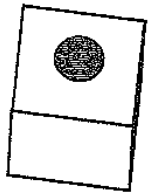


There were 9 children on the playground.
8 more children came to the playground.
Now how many children are at the playground?

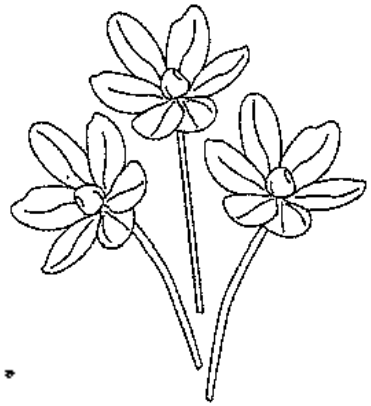
83. The animal park has 12 zebras, 25 monkeys, and some giraffes. If the total number of zebras, monkeys, and giraffes at the park is 50, how many giraffes are there?

Kim used 3 round beads to make a bracelet.
She also used some square beads.
The bracelet has 7 beads in all.
How many square beads did Kim use?





Max had 16 flowers.
He gave 5 to his mom.
How many flowers did he have left?



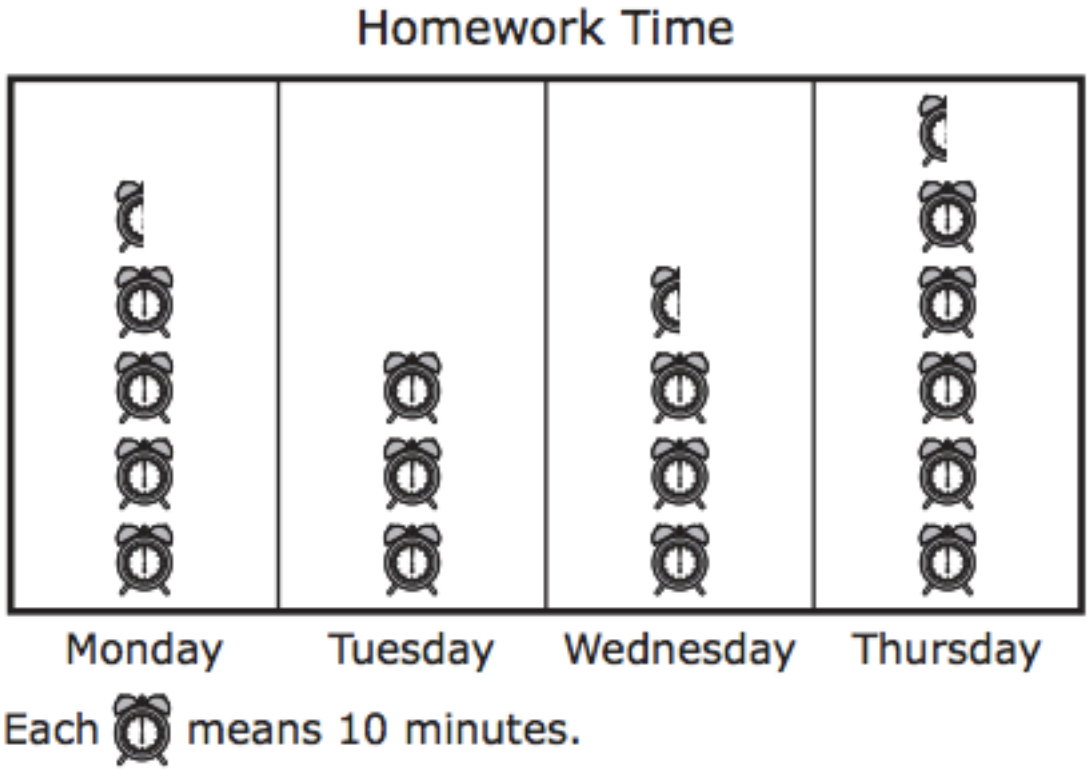
84. The grocery store had 517 jars of crunchy peanut butter and 434 jars of creamy peanut butter. How many fewer jars of creamy peanut butter than crunchy peanut butter were there?

Andrea picked 42 apples at the apple orchard. Her younger brother picked 13 apples. How many more apples did Andrea pick than her brother?

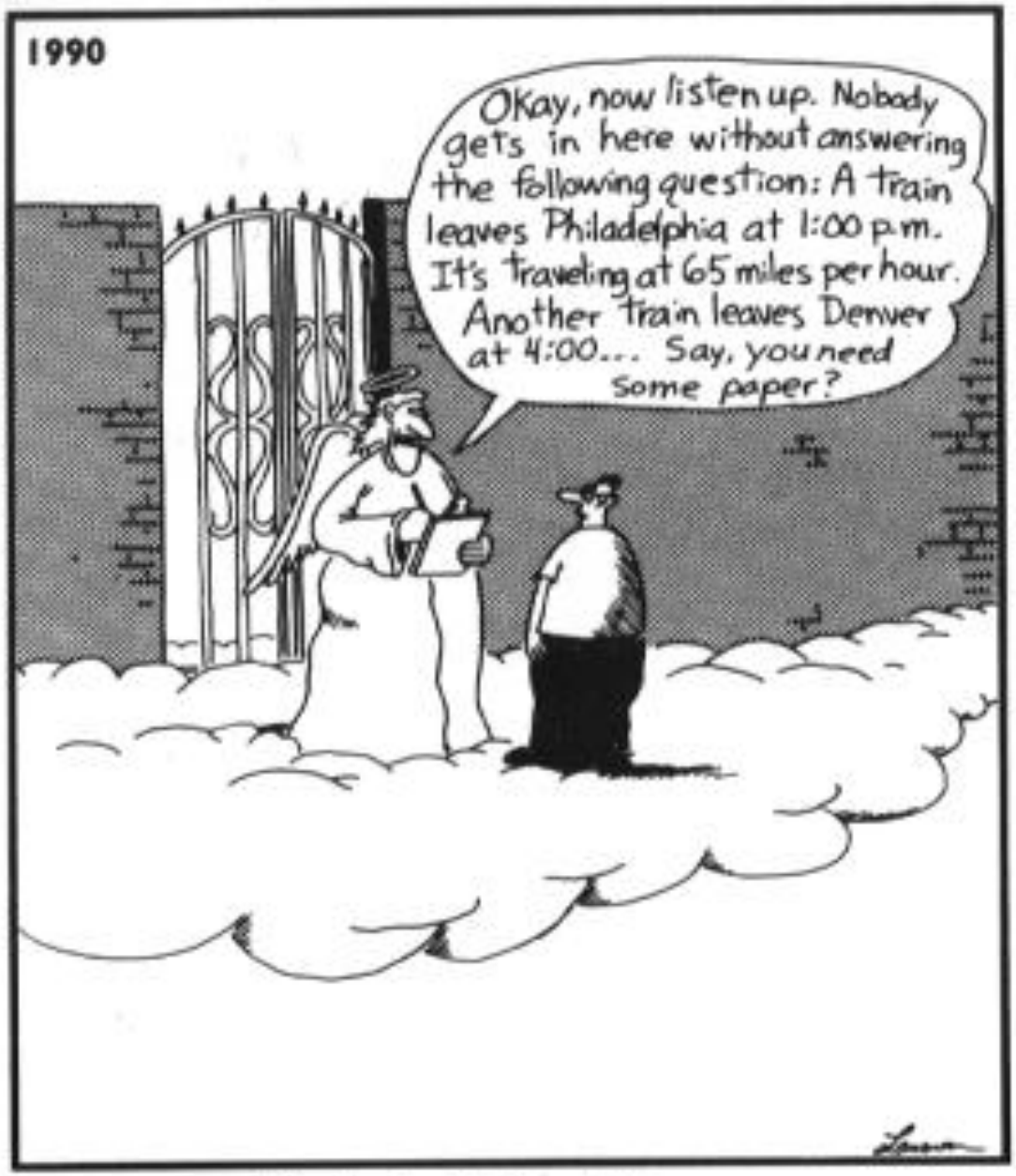
82. My friends ate 213 chocolate chips. If I eat 178 chocolate chips, how many will we have eaten in all?

60. Angelina looked in her closet and saw a container of markers. She took 42 markers out of the container and counted 88 left. How many markers were in the container when she found it in the closet?

The graph below shows the number of minutes Ryan spent doing homework during four nights.



How many minutes did Ryan spend doing homework on Tuesday and Thursday combined?



Math phobic's nightmare

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Let's Review

- What are some general word-problem solving strategies?
- What's a Combine problem?
- What's a Compare problem?
- What's a Change problem?
  - What are the two types of Change problems?
- Why should students learn problem types/ schemas?

MULTIPLICATIVE SCHEMAS

Equal Groups

- **Groups** multiplied by **number in each group** for a **product**

  G x **N** = **P**

- Examples
  - Mark has **4** bags of apples. There are **6** apples in each bag. How many apples does Mark have altogether?
    - **4** x **6** = **?** **MULTIPLICATION**
  - Mark has **24** apples. He wants to share them equally among his **4** friends. How many apples will each friend receive?
    - **4** x **?** = **24** **PARTITION DIVISION**
  - Mark has **24** apples. He put them into bags containing **6** apples each. How many bags did Mark use?
    - **?** x **6** = **24** **MEASUREMENT DIVISION**

Comparison

- **Set** multiplied by a number of **times** for a **product**

  S x **T** = **P**

- Examples
  - Jill picked **6** apples. Mark picked **4** times as many apples as Jill. How many apples did Mark pick?
    - **6** x **4** = **?** **MULTIPLICATION**
  - Mark picked **24** apples. He picked **4** times as many apples as Jill. How many apples did Jill pick?
    - **?** x **4** = **24** **PARTITION DIVISION**
  - Mark picked **24** apples, and Jill picked **6** apples. How many times as many apples did Mark pick as Jill did?
    - **6** x **?** = **24** **MEASUREMENT DIVISION**

Combinations

- **Set** multiplied by a **set** for a **product**

  S1 x **S2** = **P**

- Examples
  - Sam bought 4 pairs of pants and 3 jackets, and they can all be worn together. How many different outfits consisting of a pair of pants and a jacket does Sam have?
    - **4** x **3** = **?**

Multiplicative

(groups) × (number in each group) = (product)

(set) × (times) = (product)

Ms. Thompson sold 6 cartons of cherries at the Farmers' Market on Saturday. Each carton holds 25 cherries. How many cherries did she sell?

Niah packed 24 dozen candles into 3 boxes. She packed an equal number of candles into each box. How many candles did Niah pack into each box?

140. Tammy has 7 trophies. Her mom has twice as many trophies. How many trophies does Tammy's mom have?

Each of 16 students in a class made a poetry book. Each book contained 24 poems. How many poems are in 16 books?

165. A fish tank holds 20 gallons of water. If you have to fill up the fish tank using a 5-gallon bucket, how many bucketfuls will you need?

If a scuba diver could carry 36 coins to the surface in one trip, how many trips would it take to carry 108 coins?

119. Chairs often come with 4 legs. If you have 6 chairs, how many total legs will there be?

165. A fish tank holds 20 gallons of water. If you have to fill up the fish tank using a 5-gallon bucket, how many bucketfuls will you need?

Isaiah put 301 floor tiles in 7 rows. Each row had the same number of tiles. How many tiles did Isaiah put in each row?

144. A basket of 28 muffins was shared equally among 4 people. How many muffins did each person get?

142. At the store, lollipops are sold in packages of 4. How many packages would you need to buy to have 24 lollipops?

117. Eileen paid $12 for 4 books. How much did each book cost?

Zenobia put 3 large pictures and 4 small pictures on each page of a photo album. What is the total number of large pictures and small pictures on 9 pages of the album?

4. Janice buys 3 bags of 100 pretzels and 2 bags of 10 pretzels. How many total pretzels does she buy?

There are 27 large T-shirts and 15 small T-shirts in a box. Each T-shirt costs $9. What is the cost of all the T-shirts in the box?

Jonas has 18 packages of gum that each contain 5 pieces. Jonas gives 16 pieces of gum to his friends. Which number sentence shows one way to find the number of pieces of gum Jonas has left?

A $18 + 16 + 5 = 39$

B $18 \times 5 - 16 = 74$

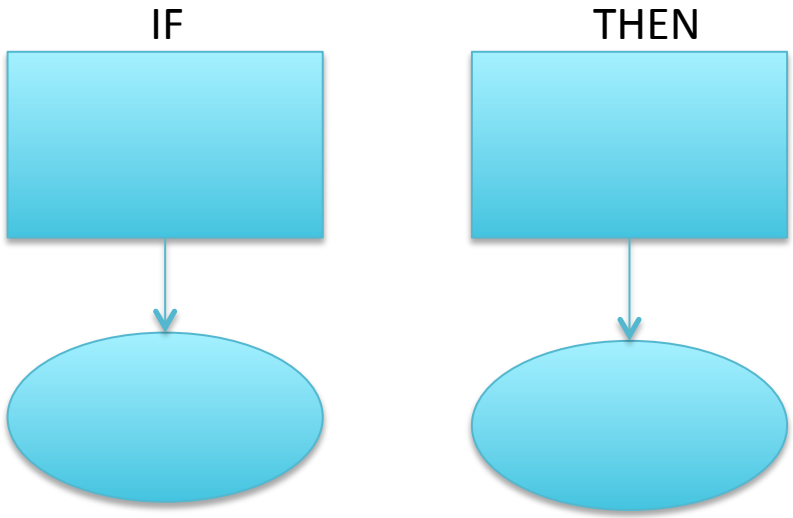
C $18 + 16 - 5 = 29$

D $18 \times 5 + 16 = 106$

Ratios and Proportions

FOPS

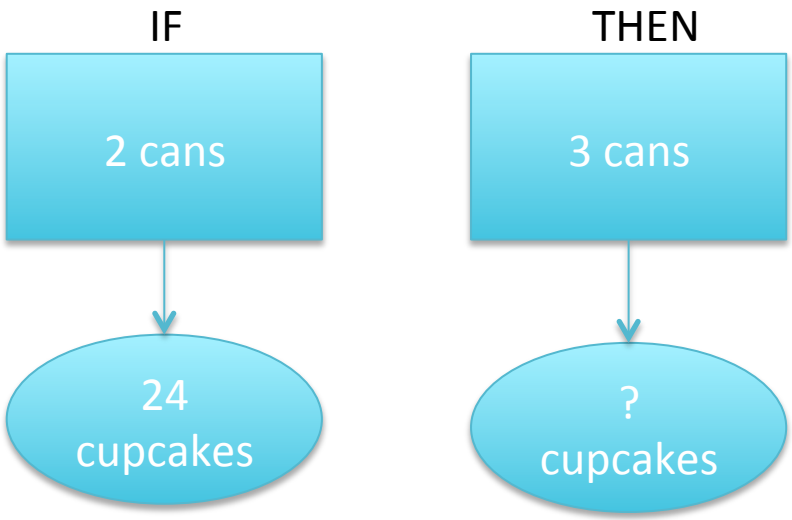
Find the problem type
Organize the information using a diagram
Plan to solve the problem
Solve the problem



Sylvia used 2 cans of icing to ice 24 cupcakes. How many cupcakes can she ice with 3 cans of icing?

Jitendra & Star (2011)

Ratios and Proportions



Sylvia used 2 cans of icing to ice 24 cupcakes. How many cupcakes can she ice with 3 cans of icing?

$$\frac{\text{2 cans}}{\text{24 cupcakes}} = \frac{\text{3 cans}}{\text{? cupcakes}}$$

36 cupcakes

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Let's Review

- What's a strategy you would use for solving multiplicative problem types?
- What's an Equal Groups problem?
- What's a Comparison problem?
- What's a Combinations problem?

A LOOK AT HIGH-STAKES ITEMS

Ramón has a total of 815 sheep in two fields. He has 348 sheep in one of the fields. How many sheep does Ramón have in the other field?

F 533

G 577

H 377

J 467

There are 27 large T-shirts and 15 small T-shirts in a box. Each T-shirt costs $9. What is the cost of all the T-shirts in the box?

A $378

B $51

C $58

D $458

Mrs. Lanier saved $617 in January. In February she spent $249 of the money she had saved. She saved $291 more in March. Which number sentence can be used to find the amount of money Mrs. Lanier had at the end of March?

F $617 + 249 - 291 = \square$

G $617 + 249 + 291 = \square$

H $617 - 249 - 291 = \square$

J $617 - 249 + 291 = \square$

Fernando's car weighs 2 tons. Keith's car weighs 3,285 pounds. What is the difference between these two weights in pounds?

Record your answer and fill in the bubbles on your answer document. Be sure to use the correct place value.

Isaiah put 301 floor tiles in 7 rows. Each row had the same number of tiles. How many tiles did Isaiah put in each row?

F 43

G 41

H 42

J 40

Isabel has 745 strawberries. She separated the strawberries into 5 equal groups. How many strawberries are in 2 of the groups?

F 202, because $745 \div 5 = 101$ and $101 \times 2 = 202$

G 282, because $745 \div 5 = 141$ and $141 \times 2 = 282$

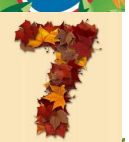
H 298, because $745 \div 5 = 149$ and $149 \times 2 = 298$

J 290, because $745 \div 5 = 145$ and $145 \times 2 = 290$








Mrs. Zapata paid a total of $8.17 to mail three packages.

- She paid $2.77 to mail the first package.
- She paid $3 to mail the second package.

How much did Mrs. Zapata pay to mail the third package?

F $3.60

G $2.40

H $6.37

J Not here

A water dispenser contains 512 fluid ounces of water. What is the total number of 8-fluid-ounce cups of water that can be filled from the dispenser?

F 611

G 64

H 4,096

J 61

An individual computer lab session at a school is 24 minutes long. On Monday 313 students each completed a session at the computer lab. What is the total number of minutes that all these students spent in the computer lab on Monday?

F 337 min

G 7,402 min

H 1,878 min

J Not here

There are 176 slices of bread in 8 loaves. If there are the same number of slices in each loaf, how many slices of bread are in 5 loaves?

F 110

G 173

H 100

J 163

On Tuesday morning a school cafeteria served 16 gallons of orange juice during breakfast. How many cups are in 16 gallons?

A 256 cups

B 64 cups

C 2,048 cups

D 128 cups

Mrs. Stephens drove through a total of 36 intersections on her way home from work last week. At 4 of every 16 intersections, Mrs. Stephens had to stop for a red light before she could drive through. At how many intersections did Mrs. Stephens have to stop for a red light?

A 3

B 9

C 24

D 12

A sea turtle made 460 dives in 12 hours. At this rate, how many dives did the sea turtle make in 3 hours?

F 153

G 115

H 120

J 165

Mrs. Rodríguez will make name tags for each of the 45 choir members and 30 orchestra members. The materials for each name tag cost $0.44. What is the total cost of the materials Mrs. Rodríguez will use to make these name tags?

F $33.00

G $75.00

H $58.20

J $49.80

Matt has answered $\frac{20}{25}$ of the questions on a test. What percentage of the test questions has Matt answered?

F 20%

G 45%

H 95%

J 80%

Yael worked out at a gym for 2 hours. Her workout consisted of stretching for 21 minutes, jogging for 45 minutes, and lifting weights for the remaining amount of time. What percentage of Yael's workout was spent lifting weights?

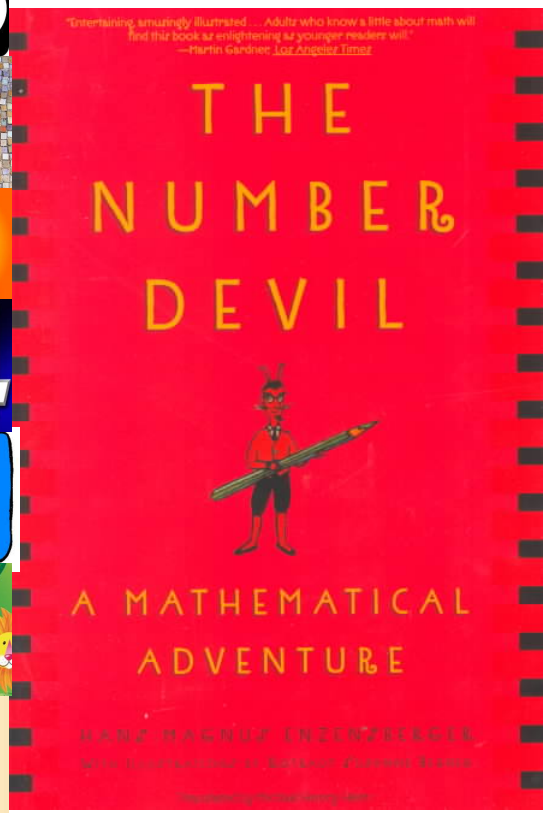
F 55%

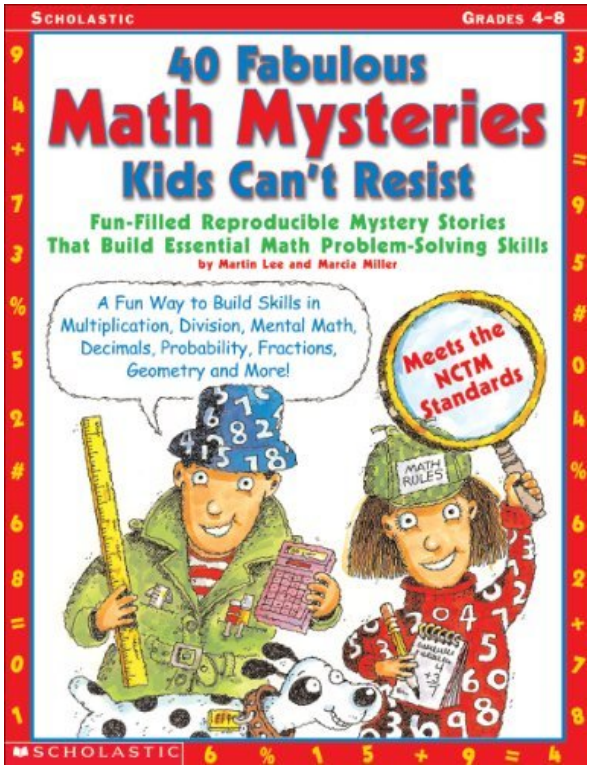
G 45%

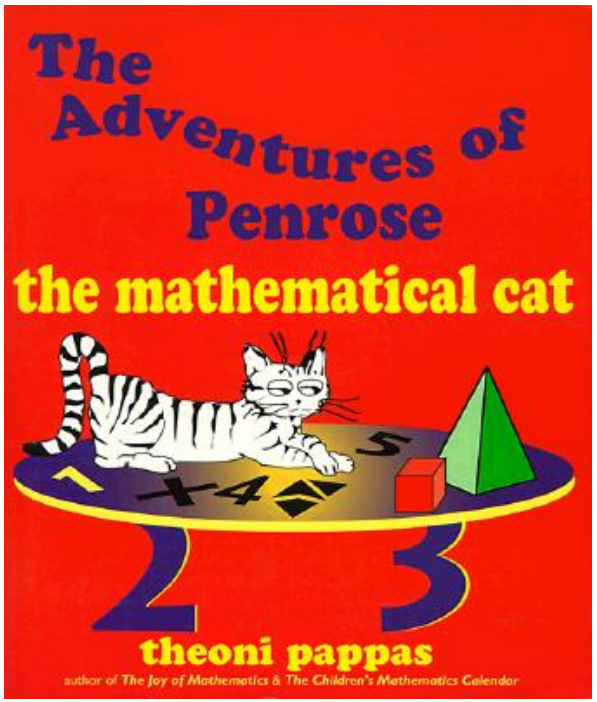
H 66%

J 54%

Good Problem Solving Books







Thank you!

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