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Spatial Reasoning Cognitive Interviews: Quantitative Data Analyses

Technical Report 20-08

Spatial Reasoning: Cognitive Interviews Methods and Quantitative Data Analyses

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Fall 2020

Published by
Southern Methodist University
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This research was supported by The National Science Foundation (Grant #1721100). Opinions expressed herein do not necessarily reflect those of The National Science Foundation or individuals within.

Acknowledgements: We would like to thank Moraima Castro-Faix for assistance with the analysis of strategy alignment, and the Research in Mathematics team for their data collection and analysis assistance.

Abstract

The purpose of this technical report is to describe the quantitative analysis from the Spatial Reasoning (SR) Cognitive Interviews (CIs) that were conducted as part of the Measuring Early Mathematics Reasoning Skills (MMaRS) project aimed for grades K-2. The CIs serve as one source of data for empirically recovering the hypothesized SR learning progression. This report details the methods used to analyze the correctness of student responses to cognitive interview protocol items and the fidelity of administration data. More details about the SR cognitive interview protocol development can be found in the Spatial Reasoning Cognitive Interview Protocol Development technical report (Tech. Rep. No.20-07). Details about the interview administration can be found in the Spatial Reasoning Cognitive Interview Administration technical report (Tech. Rep. No. 20-23).

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Spatial Reasoning Cognitive Interviews: Quantitative Data Analyses

Introduction

The purpose of this report is to describe the quantitative analyses conducted from the Spatial Reasoning (SR) Cognitive Interviews (CIs) of the Measuring Early Mathematics and Reasoning Skills (MMaRS) project. Based on the hypothesized SR Learning Progression (LP), we developed CI protocols and implemented those to inform the conceptualization and empirical recovery of the LP. See the Spatial Reasoning Cognitive Interview Protocol Development (Tech. Rep. No. 20-07) and Spatial Reasoning Cognitive Interview Administration (Tech. Rep. No. 20-23) technical reports for development and administration details. This report details the analyses of quantitative data to inform our overall research questions and later LP reconciliation.

Research Questions

We designed the cognitive interviews to address four research questions related to empirically evaluating the SR learning progression by eliciting and studying students' Knowledge, Skills, and Abilities (KSAs). We included detailed sub-questions within each overarching research question. This report details the results of the analysis to address Research Questions 1.3, 2.2, and 2.3. To find information on the other research questions, see the Spatial Reasoning Cognitive Interviews: Qualitative Data Analyses technical report (Tech. Rep. No. 20-21).

RQ 1: Developmental Appropriateness

- 1.1 Do the entry and exit KSAs align with teachers' expectations of pre-requisite and target skills?
- 1.2 Does teachers' frequency of teaching KSA align with progression?
- 1.3 Does student performance and engagement indicate floor or ceiling effects that align with entry and exit KSAs?

RQ 2: Ordering

- 2.1 Are teachers' perceptions of the appropriateness aligned with the hypothesized order?
- 2.2 Do students demonstrate increasingly sophisticated reasoning aligned with the hypothesized ordering?
- 2.3 Do students appear comfortable with tasks and task elements?

RQ 3: Conceptions

- 3.1 Do students demonstrate reasoning that is consistent with the hypothesized conceptions?
- 3.2 What misconceptions and/or errors do students make? Is there a pattern leading to greater competence?

RQ 4: Interconnectedness

4.1 In what ways are students' KSAs interconnected?

4.2 In what ways do prior KSAs impact students' responses?

Table 1 describes the data used by research question.

Table 1

Data use by research question

Research Question	Data Use
1	
1.1	Teacher Survey Data
1.2	Teacher Survey Data
1.3	Quantitative Data (p-values)
2	
2.1	Teacher Survey Data
2.2	Quantitative Data (c-prop)
2.3	Fidelity Data
3	
3.1	Quantitative and Qualitative Data
3.2	Classification or Incorrect CI Responses and Qualitative Data
4	
4.1	Qualitative Data
4.2	Qualitative Data

Data Processing

Data gathered in cognitive interviews was used for the quantitative analysis described herein. For information about the creation of cognitive interview items and their alignment to the learning progression, please see the Spatial Reasoning Cognitive Interview Protocol Development technical report (Tech. Rep. No. 20-07). The primary data sources included the student interview videos, transcripts, and fidelity observation forms. After data collection, the interview team securely stored the data in a locked filing cabinet for data processing. A team member sorted the interview materials and securely uploaded video and audio files to BOX.

We added the students' gestures into the transcript document to reduce cognitive load and better understand students' reasoning due to spatial reasoning tasks' visual and multidimensional nature. A group of internal/external team members watched the videos, segmented the file by subcomponent, removed any non-mathematical conversation, and added the student gestures. Interviewers' gestures were included if they added meaning to the student's response. Details of this process and qualitative analysis of gestures can be found in the Spatial Reasoning Cognitive Interviews: Qualitative Data Analyses technical report (Tech. Rep. No. 20-21); these transcripts, including non-verbal gestures and action, were used when needed for quantitative analysis.

To better understand the learning progression's ordering and conceptualization, we scored the cognitive interview items for correctness. Because some subcomponents included multiple content questions, we implemented scoring rules, which are described in Table 2. We used the scoring rules to understand the ordering of the subcomponents within a core concept. The scoring rules also included alignment scores for some items (see Table 3). We scored students as aligned if they performed the skill using the intended strategy and entered scores by targeted learning goal into a Microsoft Excel spreadsheet. See the Spatial Reasoning Cognitive Interview Protocol Development (Tech. Rep. No. 20-07) for a full conceptualization of the LP, including Targeted Learning Goals (TLGs), Core Concepts (CCs), and subcomponents.

Table 2

Scoring rules for SR items

Subcomponent	Scoring Rule
Within 1	
A.1.a	Sorted correctly with any sorting scheme.
A.1.b (2D)	Score correct if student obtains majority (> 50%) of shapes
A.1.c (2D)	Score correct if student obtains majority (> 50%) of shapes
A.1.d (2D)	Score correct if student correctly describes 4/6 shapes with any attribute
A.1.b (3D)	Score correct if student obtains majority (> 50%) of shapes
A.1.c (3D)	Score correct if student obtains majority (> 50%) of shapes
A.1.d (3D)	Score correct if student correctly describes 75% of shapes using any attribute.
Within 2	
A.2.a	Score correct if majority (>50%) correct.
A.2.b	Score correct if majority (>50%) correct.
A.2.c (2D)	Score correct if task 1 correct.
A.2.d (3D Shape)	Score correct if task 1 correct.
A.2.d (3D Figure)	Score correct if task 1 correct.
A.2.e. Fold	Correct/Incorrect
A.2.e Fold & Punch	Correct/Incorrect
Within 3	
A.3.a (2D)	Correct/Incorrect
A.3.a (3D)	Correct/Incorrect
A.3.b (2D)	Correct if Puzzle 2 correct
A.3.b (3D)	Correct if Task 1 correct
A.3.c (2D)	Correct if at least 2 shape compositions
A.3.c (3D)	Correct if at least 2 shape compositions
A.3.d	Correct if task 2 correct
A.3.e	Correct if both correct
A.3.f	Correct/Incorrect
A.3.g	Correct if both correct
Between 5	
B.5.a	Correct/Incorrect
B.5.b	Correct if 2/3 places correctly
B.5.c	**Not Scorable (Use qualitative analysis)**

Subcomponent	Scoring Rule
Between 6	
B.6.a	Correct/Incorrect
B.6.b	Correct if both correct
B.6.c	Correct/Incorrect
B.6.d	Correct if ¾ of items properly drawn.
B.6.e	Correct if both correct
B.6.f	Correct/Incorrect
B.6.g	Correct/Incorrect
B.6.h	Correct/Incorrect
Between 7	
B.7.a	Correct/Incorrect
B.7.b	Correct/Incorrect
B.7.c	Correct/Incorrect
B.7.d	Correct/Incorrect
B.7.e (Task 1)	Correct if 4/6 items placed correctly
B.7.e (Task 2)	Correct if 3/5 items placed correctly

Table 3

Alignment scoring for select SR items

Subcomponent	Alignment Rule
Within	
A.1.a	Student does not sort by dimension.
A.2.a	Student models the translation.
A.2.c (2D)	Student models flip/turn.
A.2.d (3D Shape)	Student does not transform shape.
A.2.d (3D Figure)	Student responds comparing the structure of figures.
A.3.a (3D)	Student mentally rotates
A.3.d	Student recognizes the embedded figure.
A.3.f	Student decomposes the figure.
A.3.g	Student iterates the figure.
Between	
B.5.c	Student uses positional language.
B.6.a	Student uses scale or 1:1 correspondence.
B.6.d	Student draws from an aerial view.

Fidelity Form

During the cognitive interviews, observers were also required to make notes about the fidelity of administration of the protocols using specific Fidelity of Administration forms. In the Fidelity of Administration form, observers entered responses whether the interviewer reworded or repeated the question and the student's comfort with the task and the manipulatives. See Figure 1 for a sample Fidelity of Administration form.

Figure 1

Fidelity of Administration Sample Form

LP: Spatial Reasoning – Within 12

Skill Code	Did the interviewer reword the **content** question? 0 - No 1 - Yes	Did the interviewer repeat the **content** question? 0 - No 1 - Yes	Did the interviewer reword the **reasoning** question? 0 - No 1 - Yes	Did the interviewer repeat the **reasoning** question? 0 - No 1 - Yes	Did the student seem comfortable with the materials? NA 0 - No 1 - Yes	How comfortable did the student appear with the task? 0 Not comfortable 1 2 3 Very comfortable
SR.A.1.a	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.1.b 2D	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.1.c/d 2D	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.1.b 3D	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.1.c/d 3D	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.a/b	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.c 2D	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.d 3D_shape	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.d 3D_figure	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.e _Fold	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.e_Punch	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3

Quantitative Data Analyses

In this section, we describe the analytical approaches used to address each research question.

RQ 1.3: Developmental Appropriateness

In addition to scoring individual items for correctness, we also scored items to better understand possible floor and ceiling effects of certain subcomponents. We calculated the *p*-values, or the probability, that a student correctly answered the subcomponent item, incorporating multi-part items where necessary. These proportions were averaged across items to provide an overall p-value that was reflective of performance across the multiple items. High p-values can indicate ceiling effects within a grade, while low p-values could indicate floor effects. We report p-values by subcomponent and grade level in Appendix A.

RQ 2.2: Ordering

We calculated several classification accuracy statistics to evaluate the appropriateness of the subcomponent ordering within a core concept. These include the false positive rate, false discovery rate, and the proportion of students who correctly answered the subcomponent items hypothesized to be more difficult and incorrectly answered those that were designed to be less difficult (c-prop). All statistics provide information regarding subcomponent ordering. High proportions may indicate the improper ordering of the subcomponents. Tables describing the following analyses are located in Appendix B.

False positive rate. A conditional probability, the false positive rate is the proportion of students who correctly answered the hypothesized more difficult subcomponent item (M+1) out of the

total number of students who incorrectly answered the hypothesized less difficult subcomponent item (M), or c/(a + c).

False discovery rate. A conditional probability, the false discovery rate is the proportion of students who incorrectly answered the hypothesized less difficult subcomponent item (M) out of the total number of students who correctly answered the hypothesized more difficult subcomponent item (M+1), or c/(c + d).

C-Prop. The c-prop is the proportion of students who correctly answered the hypothesized more difficult subcomponent item (M+1) *and* incorrectly answered the hypothesized less difficult subcomponent item (M) out of the total number of students, or c/(a + b + c + d); see Figure 2.

Figure 2

C-Prop: Classification Accuracy Statistics

		M		Totals
		Incorrect	Correct	
M + 1	Incorrect	a	b	a + b
	Correct	c	d	c + d
	Totals	a + c	b + d	a + b + c + d

RQ 2.3: Fidelity of Administration

To ensure the fidelity of interview protocols, measure how often a protocol item was reworded or repeated by the interviewer, and quantify the observers' perceived comfort of the student with the question and with the manipulatives, observers recorded such information on a Fidelity of Administration form. We analyzed these data by reporting the frequencies and percentages of these events across the sample and by grade level. Tables describing these results are located in Appendix C.

Results of the Quantitative Analyses

In this section, we describe the results of quantitative analyses by research question.

RQ 1.3: Developmental Appropriateness

In this section we describe the results for the developmental appropriateness of subcomponents by illustrating the findings for the Shape CC of the Within Objects TLG. The results for the Transformations and Composition/Decomposition CCs of the Within Objects TLG, all Between Objects TLG CCs, and alignment scoring are located in Appendix A. Table 6 provides the difficulty indices (p-values) for each subcomponent across the sample and by grade level. Higher values indicate less difficult items. For example, A.1.a was determined to be more difficult than

A.1.b 2D because the p-value is higher for the latter. Table 4 provides the difficulty indices when we accounted for alignment of the subcomponent.

Table 4

Difficulty indices overall and by grade level: Shape Correct/Aligned

	K	N	1	N	2	N	Overall	N
A.1.a	0.333	6	0.600	5	0.000	5	0.313	16
A.1.b 2D	0.563	6	0.650	5	0.725	5	0.641	16
A.1.c 2D	0.515	6	0.582	5	0.636	5	0.574	16
A.1.d 2D	0.228	6	0.700	5	0.900	5	0.585	16
A.1.b 3D	0.690	6	0.486	5	0.829	5	0.670	16
A.1.c 3D	0.229	6	0.100	5	0.300	5	0.211	16
A.1.d 3D	0.042	6	0.150	5	0.700	5	0.281	16

RQ 2.2: Ordering

In this section, we report the results for the ordering of subcomponents in the Shape CC of the Within Objects TLG. The results for Transformations and Composition/Decomposition CCs of the Within Objects TLG and Between Objects TLG CCs are located in Appendix B. Table 5 describes the false positive proportions across the Shape CC, in which higher proportions indicate issues with ordering. False positives refer to the proportion of students who correctly answered the hypothesized more difficult subcomponent items correctly out of the total number of students who incorrectly answered the hypothesized less difficult subcomponent items. For example, out of 11 students who incorrectly answered A.1.a, 64% correctly answered A.1.b (2D). The sample in each column represents the total number of students who incorrectly answered the hypothesized less difficult subcomponent item. A limitation of false positive rates is that it only provides information about the students who incorrectly answered the hypothesized easier subcomponent.

Table 5

False positive rates correct/aligned for shape core concept

	A.1.b 2D	N	A.1.c 2D	N	A.1.d 2D	N	A.1.b 3D	N	A.1.c 3D	N	A.1.d 3D	N
A.1.a	0.636	11	0.909	11	0.091	11	0.727	11	0.182	11	0.091	11
A.1.b 2D			0.600	5	0.400	5	0.600	5	0.000	5	0.000	5
A.1.c 2D					0.200	5	0.200	5	0.000	5	0.000	5
A.1.d 2D							0.571	7	0.000	7	0.000	7
A.1.b 3D									0.000	5	0.000	5
A.1.c 3D											0.133	15

Table 6 describes the false discovery rate for the Shape CC of the Within Objects TLG. False discovery rate refers to the proportion of students who incorrectly answered the hypothesized

less difficult subcomponent item out of the total number of students who correctly answered the hypothesized more difficult subcomponent item. Similar to the false positive proportion, higher proportions indicate an issue with ordering. For example, out of the 11 students who correctly answered A.1.b 2D, 64% of students incorrectly answered A.1.a. The sample size in each column represents the total number of students who correctly answered the hypothesized more difficult subcomponent item. A limitation of the false discovery rate is that it only provides information about those who correctly answered the hypothesized more difficult subcomponent.

Table 6

False discovery rates correct/aligned for shape core concept

	A.1.b 2D	N	A.1.c 2D	N	A.1.d 2D	N	A.1.b 3D	N	A.1.c 3D	N	A.1.d 3D	N
A.1.a	0.636	11	0.909	11	0.111	9	0.727	11	0.500	1	0.333	3
A.1.b 2D			0.273	11	0.222	9	0.273	11	0.000	1	0.000	3
A.1.c 2D					0.111	9	0.091	11	0.000	1	0.000	3
A.1.d 2D							0.364	11	0.000	1	0.000	3
A.1.b 3D									0.000	1	0.000	3
A.1.c 3D											0.667	3

Table 7 describes the proportion of students who correctly answered the hypothesized more difficult subcomponent item and incorrectly answered the hypothesized less difficult subcomponent items out of the total number of students. Higher proportions indicate an issue with ordering. For example, out of the 16 interviewed students, 19% incorrectly answered A.1.b 2D but correctly answered A.1.c 2D. The sample sizes are indicated of all students who were presented both tasks. The c-proportion circumvents the issues of using the false positive and false discovery rates by accounting for students who incorrectly answered the hypothesized easier subcomponent and correctly answered the hypothesized more difficult subcomponent.

Table 7

C-proportion corrected/aligned for shape core concept

	A.1.b 2D	N	A.1.c 2D	N	A.1.d 2D	N	A.1.b 3D	N	A.1.c 3D	N	A.1.d 3D	N
A.1.a	0.438	16	0.625	16	0.0625	16	0.500	16	0.125	16	0.125	16
A.1.b 2D			0.188	16	0.125	16	0.188	16	0.000	16	0.000	16
A.1.c 2D					0.063	16	0.063	16	0.000	16	0.000	16
A.1.d 2D							0.250	16	0.000	16	0.000	16
A.1.b 3D									0.000	16	0.000	16
A.1.c 3D											0.125	16

RQ 2.3: Fidelity of Administration

During the cognitive interviews, observers used Fidelity of Administration forms to collect information about the fidelity of data collection. For each subcomponent item presented, observers entered data for the four questions listed below, with reword and repeat fields for content and reasoning questions recorded separately.

1. Did the interviewer reword the (Content and Reasoning) question?
2. Did the interviewer repeat the (Content and Reasoning) question?
3. Did the student seem comfortable with the manipulatives?
4. How comfortable did the student appear with the task?

In this section, we report the analyses of Fidelity of Administration data by subcomponent and grade level for each TLG. These data are presented in Appendix C.

Within Objects Spatial Reasoning TLG

Rewording of Content Questions. For kindergarten participants, 14 out of 21 total content questions needed rewording for clarification or communication purposes. The interviewer needed to reword each content question for the six kindergarten students between 0% to 58% of the time. For first-grade participants, 11 out of 21 total interview content questions needed rewording for clarification or communication purposes. The percentage of times the interviewer needed to reword each content question for the five first-grade students ranged from 0% to 100%. For second-grade participants, 13 out of 21 total interview content questions needed rewording for clarification or communication purposes. The percentage of times the interviewer needed to reword each content question for the five second-grade students ranged from 0% to 60%. For 16 participants from all grade levels combined, the percentage of times the interviewer needed to reword a content question for a subcomponent item ranged between 0% and 69%, and for eight subcomponent items—SR.A.1.c/d (3D), SR.A.2.c (2D shape), SR.A.2.d (2D shape), SR.A.2.d (3D figure), SR.A.2.e (Fold), SR.A.2.e (Fold & Punch), SR.A.3.b (2D), and SR.A.3.f—content question were reworded above 25% (i.e., 31%, 38%, 40%, 44%, 44%, 38%, 69%, and 47% respectively).

Rewording of Reasoning Questions. For kindergarten students, 12 out of 21 total interview reasoning questions needed rewording for clarification or communication purposes. The percentage of times the interviewer reworded reasoning questions for the six kindergarten students ranged from 0% to 40%. For first-graders, eight out of 21 total interview reasoning questions needed rewording for clarification or communication purposes. The percentage of times the interviewer needed to reword each reasoning question for the five first-grade students ranged from 0% to 60%. For second-graders, seven out of 21 total interview reasoning questions needed rewording for clarification or communication purposes. The percentage of times the interviewer required to reword each reasoning question for the five second-grade students ranged from 0% to 40%. For 16 participants from all grade levels combined, the percentage of times the interviewer needed to reword a reasoning question for a subcomponent item ranged between 0%

and 31%. Three subcomponent items—SR.A.1.b (2D), SR.A.1.b (3D), and SR.A.2.e (Fold & Punch)—were reworded above 25% (i.e., 31%, 31%, and 29% respectively).

Repeating Content Questions. For kindergarten students, 14 out of 21 interview content questions were repeated for clarification or communication purposes. The percentage of times the interviewer repeated the content questions for the six kindergarten students ranged from 0% to 33% (for two out of six students). Interviewers repeated four out of 21 total interview content questions for clarification or communication purposes for first-graders. The percentage of times the interviewer repeated the content questions for the five first-grade students ranged from 0% to 20% (for one out of five students). For second-graders, ten out of 21 interview content questions were repeated for clarification or communication purposes. The percentage of times the interviewer repeated the content questions for the five second-grade students ranged from 0% to 40% (for two out of five students). For the 16 participants from all grade levels combined, the percentage of times the interviewer needed to repeat a content question for a subcomponent item ranged between 0% and 25%, and none of the subcomponent content questions were repeated above 25%.

Repeating Reasoning Questions. For kindergarten students, 11 out of 21 total interview reasoning questions were repeated for clarification or communication purposes. The percentage of times the interviewer repeated the reasoning questions for the six kindergarten students ranged from 0% to 40%. For first-graders, nine out of 21 total interview reasoning questions were repeated for clarification or communication purposes. The percentage of times the interviewer repeated the reasoning questions for five first-grade students ranged from 0% to 60%. For second-graders, six out of eleven total interview reasoning questions were repeated for clarification or communication purposes. The percentage of times the interviewer needed to repeat reasoning questions for the five second-grade students ranged from 0% to 20%. For the 16 participants across grade levels, the percentage of times the interviewer needed to repeat a reasoning question for a subcomponent item ranged between 0% and 33%. Three subcomponent items—SR.A.1.b (2D), SR.A.2.d (2D Shape), and SR.A.2.d (3D Figure)—were reworded above 25% (i.e., 31%, 33%, and 27% respectively).

Comfort with manipulatives. Students across grades seemed completely comfortable with the manipulatives provided during the interviews for various subcomponent items. The percentage of comfort with manipulatives was 100% for all subcomponents except two—SR.A.3.b 2D and SR.A.3.c 3D—in which comfort with manipulatives dropped to 94%. The minimum comfort level was recorded at 80% for two subcomponentslSR.A.3.b 2D and SR.A.3.c 3D—among first- and second-grade students. Kindergartners were comfortable with 100% of the manipulatives presented that were associated with ten subcomponents.

Comfort with tasks (Range 0-3points). For all subcomponents, the average score for comfort with tasks was 2.79 across grade levels. The lowest comfort score of 2.50 was recorded for subcomponent A.1.a across all grades. For a grade-wide comparison of comfort with tasks, the lowest score was 2.65 for first-grade students, and the highest was 2.87 for second-graders. Overall, there was not much difference in the comfort score between grades, with averages of kindergarten at 2.85, first-grade at 2.65, and second-grade at 2.87.

Between Objects Spatial Reasoning TLG

Rewording of Content Questions. For kindergarten participants, 14 out of 17 total content questions needed rewording for clarification or communication purposes. The interviewer needed to reword each content question for the six kindergarten students between 0% to 67% of the time. For first-grade participants, eight out of 17 total interview content questions needed rewording for clarification or communication purposes. The percentage of times the interviewer needed to reword each content question for the five first-grade students ranged from 0% to 40%. for second-grade participants, eight out of 17 total interview content questions needed rewording for clarification or communication purposes. The percentage of times the interviewer needed to reword each content question for the five second-grade students ranged from 0% to 40%. For 16 participants from all grade levels combined, the percentage of times the interviewer needed to reword a content question for a subcomponent item ranged between 0% and 56%, and for four subcomponent items—SR.B.5.a, SR.B.5.c, SR.B.6.d, and SR.B.7.c—content question were reworded above 25% (i.e., 31%, 56%, 50%, and 38% respectively).

Rewording of Reasoning Questions. For kindergarten students, 14 out of 17 total interview reasoning questions needed rewording for clarification or communication purposes. The percentage of times the interviewer reworded reasoning questions for the six kindergarten students ranged from 0% to 83%. For first-graders, 13 out of 17 total interview reasoning questions needed rewording for clarification or communication purposes. The percentage of times the interviewer needed to reword each reasoning question for the five first-grade students, ranged from 0% to 40%. For second-graders, 12 out of 17 total interview reasoning questions needed rewording for clarification or communication purposes. The percentage of times the interviewer needed to reword each reasoning question for the five second-grade students ranged from 0% to 60%. For 16 participants from all grade levels combined, the percentage of times the interviewer needed to reword a reasoning question for a subcomponent item ranged between 6% and 63%. For seven subcomponent items—SR.B.5.a, SR.B.6.c, SR.B.6.d, SR.B.6.h, SR.B.7.b, SR.B.7.e (Task 1), and SR.B.7.e (Task 2)—reasoning questions were reworded above 25% (i.e., 44%, 31%, 31%, 63%, 31%, 40%, and 33% respectively).

Repeating Content Questions. For kindergarten students, 13 out of 17 total interview content questions were repeated for clarification or communication purposes. The percentage of times the interviewer repeated each content question for the six kindergarten students, ranged from 0% to 33%. For first-graders, six out of 17 total interview content questions were repeated for clarification or communication purposes. The percentage of times the interviewer needed to repeat each content question for the five first-grade students ranged from 0% to 40%. For second-graders, five out of 17 total interview content questions were repeated for clarification or communication purposes. The percentage of times the interviewer needed to repeat each content question for the five second-grade students ranged from 0% to 20%. For 16 participants from all grade levels combined, the percentage of times the interviewer needed to repeat a content question for a subcomponent item ranged between 0% and 25%, and none of the subcomponent item content questions were repeated above 25%.

Repeating Reasoning Questions. For kindergarten students, six out of 17 total interview reasoning questions were repeated for clarification or communication purposes. The percentage of times the interviewer repeated each reasoning question for the six kindergarten students

ranged from 0% to 33%. For first-graders, again six out of 17 total interview reasoning questions were repeated for clarification or communication purposes. The percentage of times the interviewer needed to repeat reasoning questions for the five first-grade students ranged from 0% to 20%. For second-graders, three out of 17 total interview reasoning questions were repeated for clarification or communication purposes. The percentage of times the interviewer needed to repeat reasoning questions for the five second-grade students ranged from 0% to 20%. For 16 participants from all grade levels combined, the percentage of times the interviewer needed to repeat a reasoning question for a subcomponent item ranged between 0% and 14%, and none of the subcomponent item reasoning questions were repeated above 25%.

Comfort with manipulatives. For all grades, students seemed comfortable with the manipulatives provided during the interviews for various subcomponent items. The minimum level of comfort was recorded at 83% for one subcomponent item among kindergarten students. Across all grade levels, the percentage of comfort with manipulatives was between 94% and 100% for different subcomponents. Kindergartners were comfortable with 100% of the manipulatives presented that were associated with sixteen out of seventeen subcomponents. For one subcomponent item—SR.B.7.e (Task 1)—kindergarteners' comfort level with the manipulatives was 83%; for the rest of all subcomponents, comfort for kindergarteners remained at 100%. First- and second-graders were comfortable with 100% of the manipulatives for all subcomponents.

Comfort with tasks (Range 0-3points). For all subcomponent items, the average score for comfort with tasks was 2.79 across all grades. For a grade-wise comparison of comfort with tasks, the lowest score was 2.75 for first-grade students and the highest was 2.84 for second-graders. Overall, there was almost no difference in the comfort score between all grades, with averages of kindergarten at 2.79, first-grade at 2.75, and second-grade at 2.84. The lowest comfort score was recorded for the subcomponent SR.B.5.a as 2.56 across all grades.

Conclusion

The purpose of the cognitive interviews was to provide evidence to empirically recover the spatial reasoning learning progression. We used 16 student interviews from students in grades K-2 across two schools. We collected data related to developmental appropriateness, ordering of subcomponent skills, students' conceptions of KSAs, and interconnectedness of those KSAs. The data analyses described in the current report are an important part of the validation process. In conjunction with the qualitative analyses of the cognitive interviews (Tech. Rep. No. 20-21), expert reviews, and a teacher survey (Tech. Rep. No. 20-10), these data will inform the necessary changes to the learning progression through forthcoming reconciliation.

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Appendix A – Developmental Appropriateness Tables

Within 1: Transformation Correct/Aligned

Difficulty indices overall and by grade level: Shapes

	K	N	1	N	2	N	Overall	N
A.1.a	0.667	6	1.000	5	1.000	5	0.875	16
A.1.b 2D	0.563	6	0.650	5	0.725	5	0.641	16
A.1.c 2D	0.515	6	0.582	5	0.636	5	0.574	16
A.1.d 2D	0.228	6	0.700	5	0.900	5	0.585	16
A.1.b 3D	0.690	6	0.486	5	0.829	5	0.670	16
A.1.c 3D	0.229	6	0.100	5	0.300	5	0.211	16
A.1.d 3D	0.042	6	0.150	5	0.700	5	0.281	16

Within 2: Transformation Correct/Aligned

Difficulty indices overall and by grade level: Transformation

	K	N	1	N	2	N	Overall	N
A.2.a	0.500	6	0.400	5	0.400	5	0.438	16
A.2.b	0.708	6	0.550	5	0.750	5	0.672	16
A.2.c 2D Green	0.667	5	0.800	4	0.800	4	0.750	13
A.2.c 2D Red	0.500	1	1.000	1	0.000	1	0.500	3
A.2.d 3D Shape Green	0.200	1	0.600	4	1.00	5	0.600	10
A.2.d 3D Shape Red	0.375	4	0.500	1	NA	0	0.400	5
A.2.d 3D Figure Green	0.333	2	0.400	2	0.600	4	0.438	8
A.2.d 3D Figure Red	0.375	4	0.500	2	NA	0	0.417	6
A.2.e Fold	0.600	5	0.600	5	0.600	5	0.600	15
A.2.e F & P	0.333	6	0.600	5	0.200	5	0.375	16

Within 3: Composition/Decomposition Correct/Aligned

Difficulty indices overall and by grade level: Composition/Decomposition

	K	N	1	N	2	N	Overall	N
A.3.a 2D	0.500	6	0.600	5	0.600	5	0.563	16
A.3.a 3D	0.167	6	0.200	5	0.600	5	0.313	16
A.3.b 2D Task 2	0.500	6	0.000	5	0.400	5	0.313	16
A.3.b 2D Task 1	1.000	3	1.000	2	1.000	1	1.000	6
A.3.b 2D Task 3	0.667	3	1.000	3	0.750	4	0.800	10
A.3.b 3D Task 1	0.167	6	0.200	5	0.400	5	0.313	16
A.3.b 3D Task 2	1.000	1	1.000	1	1.000	2	0.800	5
A.3.c 2D	0.667	6	1.000	5	0.600	5	0.750	16
A.3.c 3D	0.500	6	0.600	5	1.000	5	0.750	16
A.3.d Task 2	0.833	6	0.800	5	0.600	5	0.750	16
A.3.d Task 1	NA	0	NA	0	NA	0	NA	0
A.3.d Task 3a	0.333	6	0.800	5	1.000	4	0.667	15

A.3.d Task 3b	0.333	6	0.200	5	0.500	4	0.333	15
A.3.e	0.700	5	0.700	5	1.000	5	0.800	15
A.3.f	0.167	5	0.200	5	0.000	5	0.125	15
A.3.g	0.167	5	0.000	5	0.800	5	0.313	15

Between 5: Spatial Language Correct/Aligned

Difficulty indices overall and by grade level: spatial language

	K	N	1	N	2	N	Overall	N
B.5.a	0.667	6	0.600	5	0.400	5	0.563	16
B.5.b	0.778	6	1.000	5	1.000	5	0.917	16
B.5.c*	0.666	6	0.800	5	0.800	5	0.750	16

Note: * Indicates an item rated on subjective use of spatial language.

Between 6: Understanding Models and Maps Correct/Aligned

Difficulty indices overall and by grade level: Understanding Models and Maps

	K	N	1	N	2	N	Overall	N
B.6.a	0.600	5	0.200	5	0.400	5	0.400	15
B.6.b	0.917	6	1.000	5	1.000	5	0.969	16
B.6.c	0.833	6	0.600	5	0.800	5	0.750	16
B.6.d	0.333	6	0.400	5	1.000	5	0.563	16
B.6.e	0.333	6	0.400	5	0.800	5	0.500	16
B.6.f	0.667	6	1.000	5	1.000	5	0.875	16
B.6.g	0.333	6	0.600	5	0.400	5	0.438	16
B.6.h	0.333	6	0.800	5	0.800	5	0.625	16

Between 7: Perspective Taking

Difficulty indices overall and by grade level: Perspective Taking

	K	N	1	N	2	N	Overall	N
B.7.a	0.800	5	0.800	5	0.600	5	0.733	15
B.7.b	1.000	6	0.800	5	1.000	5	0.938	16
B.7.c	0.667	6	0.400	5	0.600	5	0.563	16
B.7.d	0.200	5	0.400	5	1.000	5	0.533	15
B.7.e Task 1	0.861	6	0.800	5	0.833	5	0.833	16
B.7.e Task 2*	0.900	4	NA	0	1.000	2	0.933	6

Note: * Indicates an item students were presented if the previous item met certain thresholds

Appendix B – Ordering Tables

Within 1: Shape

False positive rates for the shape core concept

	A.1.b 2D	N	A.1.c 2D	N	A.1.d 2D	N	A.1.b 3D	N	A.1.c 3D	N	A.1.d 3D	N
A.1.a	0.500	2	0.500	2	0.000	2	0.500	2	0.000	2	0.000	2
A.1.b 2D			0.600	5	0.400	5	0.600	5	0.000	5	0.000	5
A.1.c 2D					0.200	5	0.200	5	0.000	5	0.000	5
A.1.d 2D							0.571	7	0.000	7	0.000	7
A.1.b 3D									0.000	5	0.000	5
A.1.c 3D											0.133	15

False discovery rates for the shape core concept

	A.1.b 2D	N	A.1.c 2D	N	A.1.d 2D	N	A.1.b 3D	N	A.1.c 3D	N	A.1.d 3D	N
A.1.a	0.091	11	0.091	11	0.000	9	0.091	11	0.000	1	0.000	3
A.1.b 2D			0.273	11	0.222	9	0.273	11	0.000	1	0.000	3
A.1.c 2D					0.111	9	0.091	11	0.000	1	0.000	3
A.1.d 2D							0.364	11	0.000	1	0.000	3
A.1.b 3D									0.000	1	0.000	3
A.1.c 3D											0.667	3

C-Proportion for the Shape Core Concept

	A.1.b 2D	N	A.1.c 2D	N	A.1.d 2D	N	A.1.b 3D	N	A.1.c 3D	N	A.1.d 3D	N
A.1.a	0.063	16	0.063	16	0.000	16	0.063	16	0.000	16	0.000	16
A.1.b 2D			0.188	16	0.125	16	0.188	16	0.000	16	0.000	16
A.1.c 2D					0.063	16	0.063	16	0.000	16	0.000	16
A.1.d 2D							0.250	16	0.000	16	0.000	16
A.1.b 3D									0.000	16	0.000	16
A.1.c 3D											0.125	16

Within 2: Transformation

False positive rates correct/aligned for transformation core concept

	A.2.b	N	A.2.c	N	A.2.d 3D S	N	A.2.d 3D F	N	A.2.e Fold	N	A.2.e F&P	N
A.2.a	0.556	9	0.667	9	0.444	9	0.222	9	0.444	9	0.222	9
A.2.b			0.833	6	0.667	6	0.667	6	0.800	5	0.333	6
A.2.c					0.500	4	0.500	4	0.750	4	0.250	4
A.2.d 3D S							0.333	6	0.833	6	0.167	6
A.2.d 3D F									0.333	9	0.222	9
A.2.e Fold											0.500	6

False discovery rates corrected/aligned for the transformation core concept

	A.2.b	N	A.2.c	N	A.2.d 3D S	N	A.2.d 3D F	N	A.2.e Fold	N	A.2.e F&P	N
A.2.a	0.300	10	0.083	12	0.222	9	0.286	7	0.444	9	0.333	6
A.2.b			0.167	12	0.222	9	0.571	7	0.444	9	0.333	6
A.2.c					0.333	9	0.286	7	0.333	9	0.167	6
A.2.d 3D S							0.286	7	0.556	9	0.167	6
A.2.d 3D F									0.556	9	0.333	6
A.2.e Fold											0.500	6

C-proportion corrected/aligned for the transformation core concept

	A.2.b	N	A.2.c	N	A.2.d 3D S	N	A.2.d 3D F	N	A.2.e Fold	N	A.2.e F&P	N
A.2.a	0.313	16	0.375	16	0.250	15	0.125	16	0.267	15	0.125	16
A.2.b			0.313	16	0.200	15	0.250	16	0.267	15	0.125	16
A.2.c					0.067	15	0.125	16	0.200	15	0.063	16
A.2.d 3D S							0.133	15	0.333	15	0.133	15
A.2.d 3D F									0.200	15	0.133	16
A.2.e Fold											0.200	15

Within 3: Composition/Decomposition

False positive rate corrected/aligned for the composition/decomposition core concept

	A.3.a 3D	A.3.b 2D	A.3.b 3D	A.3.c 2D	A.3.c 3D	A.3.d	A.3.e	A.3.f	A.3.g	N
A.3.a 2D	0.429	0.429	0.143	0.714	0.571	0.857	0.429	0.000	0.143	7*
A.3.a 3D		0.182	0.273	0.364	0.727	0.727	0.818	0.182	0.273	11*
A.3.b 2D			0.182	0.455	0.636	0.727	0.909	0.182	0.364	11*
A.3.b 3D				0.750	0.583	0.818	0.417	0.182	0.364	11*
A.3.c 2D					0.750	1.750	0.500	0.250	0.250	4*
A.3.c 3D						0.800	0.000	0.000	0.000	5
A.3.d							1.000	0.000	0.500	4
A.3.e								0.000	0.000	8*
A.3.f									0.286	14

Note: * indicates the number is one less for comparisons with A.3.f

False discovery rate corrected/aligned for the composition/decomposition core concept

	A.3.a 3D	A.3.b 2D	A.3.b 3D	A.3.c 2D	A.3.c 3D	A.3.d	A.3.e	A.3.f	A.3.g
A.3.a 2D	0.600	0.600	0.250	0.417	0.364	0.500	0.375	0.000	0.200
A.3.a 3D		0.400	0.600	0.333	0.727	0.667	0.625	0.500	0.600
A.3.b 2D			0.400	0.750	0.636	0.667	0.750	1.000	0.800
A.3.b 3D				0.750	0.636	0.750	0.625	0.500	0.800
A.3.c 2D					0.273	0.250	0.250	0.500	0.200
A.3.c 3D						0.333	0.000	0.000	0.000

A.3.d							0.500	0.000	0.400
A.3.e								0.000	0.000
A.3.f									0.800
N	5	5	5	12	11	12	8	2	5

C-proportion correct/aligned for the composition/decomposition core concept

	A.3.a 3D	A.3.b 2D	A.3.b 3D	A.3.c 2D	A.3.c 3D	A.3.d	A.3.e	A.3.f	A.3.g	N
A.3.a 2D	0.188	0.188	0.063	0.313	0.250	0.375	0.188	0.000	0.063	16*
A.3.a 3D		0.125	0.188	0.250	0.500	0.500	0.563	0.125	0.188	16*
A.3.b 2D			0.125	0.313	0.188	0.625	0.625	0.133	0.267	16*
A.3.b 3D				0.563	0.438	0.563	0.313	0.125	0.250	16*
A.3.c 2D					0.188	0.438	0.125	0.063	0.063	16*
A.3.c 3D						0.250	0.000	0.000	0.000	16*
A.3.d							0.250	0.000	0.125	16*
A.3.e								0.000	0.000	16*
A.3.f									0.250	15

Note: * indicates where the sample size decreased to 15 items compared to A.3.f.

Between 5: Spatial Language

False positive rate correct/aligned for the spatial language core concept

	B.5.b	N	B.5.c	N
B.5.a	1.000	7	0.714	7
B.5.b			1.000	1

False discovery rate correct/aligned for the spatial language core concept

	B.5.b	N	B.5.c	N
B.5.a	0.467	15	0.417	12
B.5.b			0.083	12

C-proportion correct/aligned for the spatial language core concept

	B.5.b	N	B.5.c	N
B.5.a	0.438	16	0.313	16
B.5.b			0.063	16

Between 6: Understanding Models and Maps

False positive rate correct/aligned for the understanding models and maps core concept

	B.6.b	B.6.c	B.6.d	B.6.e	B.6.f	B.6.g	B.6.h	N
B.6.a	1.000	0.778	0.556	0.555	1.000	0.555	0.888	9

B.6.b	1.000	0.000	0.000	1.000	0.000	1.000	1
B.6.c		0.750	0.500	1.000	0.500	0.500	4
B.6.d			0.142	0.857	0.285	0.571	7
B.6.e				0.875	0.250	0.625	8
B.6.f					0.500	0.000	2
B.6.g						0.556	9

False discovery rate correct/aligned for the understanding models and maps core concept

	B.6.b	B.6.c	B.6.d	B.6.e	B.6.f	B.6.g	B.6.h
B.6.a	0.643	0.583	0.556	0.625	0.643	0.714	0.800
B.6.b		0.083	0.000	0.000	0.071	0.000	0.100
B.6.c			0.333	0.250	0.286	0.286	0.200
B.6.d				0.125	0.429	0.286	0.400
B.6.e					0.500	0.286	0.500
B.6.f						0.143	0.000
B.6.g							0.500
N	14	12	9	8	14*	7	10

Note: * Indicates one less when comparing to B.6.a

C-proportion correct/aligned for the understanding models and maps core concept

	B.6.b	B.6.c	B.6.d	B.6.e	B.6.f	B.6.g	B.6.h	N
B.6.a	0.600	0.467	0.333	0.333	0.600	0.333	0.533	15
B.6.b		0.063	0.000	0.000	0.063	0.000	0.063	16
B.6.c			0.188	0.125	0.250	0.125	0.125	16
B.6.d				0.063	0.375	0.125	0.250	16
B.6.e					0.438	0.125	0.313	16
B.6.f						0.063	0.000	16
B.6.g							0.313	16

Between 7: Perspective Taking

False positive rate for the perspective taking core concept

	B.7.b	N	B.7.c	N	B.7.d	N	B.7.e1	N	B.7.e2	N
B.7.a	0.750	4	0.750	4	1.000	3	1.000	4	1.000	1
B.7.b			0.000	1	1.000	1	1.000	1	0.000	0
B.7.c					0.429	7	1.000	7	1.000	2
B.7.d							1.000	14	1.000	6
B.7.e1									0.000	0

False discovery rate for the perspective taking core concept

	B.7.b	N	B.7.c	N	B.7.d	N	B.7.e1	N	B.7.e2	N
B.7.a	0.214	14	0.375	8	0.375	8	0.286	14	0.200	5
B.7.b			0.000	9	0.125	8	0.067	15	0.000	6
B.7.c					0.375	8	0.467	15	0.333	6
B.7.d							0.500	14	0.500	6
B.7.e1									0.000	6

C-proportion for the perspective taking core concept

	B.7.b	N	B.7.c	N	B.7.d	N	B.7.e1	N	B.7.e2	N
B.7.a	0.200	15	0.200	15	0.214	14	0.267	15	0.200	5
B.7.b			0.000	16	0.067	15	0.063	16	0.000	6
B.7.c					0.200	15	0.438	16	0.333	6
B.7.d							0.467	15	0.500	6
B.7.e1									0.000	6

Appendix C – Fidelity of Administration Tables

Sample Fidelity of Administration forms

LP: Spatial Reasoning – Within 12

Skill Code	Did the interviewer reword the **content** question? 0 - No 1 - Yes	Did the interviewer repeat the **content** question? 0 - No 1 - Yes	Did the interviewer reword the **reasoning** question? 0 - No 1 - Yes	Did the interviewer repeat the **reasoning** question? 0 - No 1 - Yes	Did the student seem comfortable with the materials? NA 0 - No 1 - Yes	How comfortable did the student appear with the task? 0 Not comfortable 1 2 3 Very comfortable
SR.A.1.a	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.1.b 2D	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.1.c/d 2D	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.1.b 3D	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.1.c/d 3D	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.a/b	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.c 2D	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.d 3D_shape	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.d 3D_figure	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.e _Fold	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3
SR.A.2.e_Punch	0 1	0 1	0 1	0 1	NA 0 1	0 1 2 3

Within: Reword by Grade

Within	Kindergarten				Grade 1				Grade 2				All Grades K-2			
Subcomponent	Need to Reword CQ (%) [KG]	KG All n reword CQ	Need to Reword RQ (%) [KG]	KG All n reword RQ	Need to Reword CQ (%) [1st]	G1 All n reword CQ	Need to Reword RQ (%) [1st]	G1 All n reword RQ	Need to Reword CQ (%) [2nd]	G2 All n reword CQ	Need to Reword RQ (%) [2nd]	G2 All n reword RQ	Need to Reword CQ (%) [ALL]	All n reword CQ	Need to Reword RQ (%) [ALL]	Total n reword RQ
SR.A.1.a	17%	1/6	17%	1/6	20%	1/5	20%	1/5	40%	2/5	0%	0/5	25%	4/16	13%	2/16
SR.A.1.b (2D)	0%	0/6	0%	0/6	0%	0/5	60%	3/5	40%	2/5	40%	2/5	13%	2/16	31%	5/16
SR.A.1.c/d (2D)	17%	1/6	0%	0/6	0%	0/5	0%	0/4	0%	0/5	0%	0/5	6%	1/16	0%	0/15
SR.A.1.b.(3D)	17%	1/6	33%	2/6	0%	0/5	40%	2/5	0%	0/5	20%	1/5	6%	1/16	31%	5/16
SR.A.1.c/d.(3D)	50%	3/6	0%	0/6	20%	1/5	0%	0/4	20%	1/5	40%	2/5	31%	5/16	13%	2/15
SR.A.2.a/b	33%	2/6	17%	1/6	20%	1/5	20%	1/5	0%	0/5	0%	0/5	19%	3/16	13%	2/16
SR.A.2.c (2D Shape)	17%	1/6	0%	0/5	60%	3/5	20%	1/5	40%	2/5	0%	0/5	38%	6/16	7%	1/15
SR.A.2.d (2D Shape)	40%	2/5	20%	1/5	60%	3/5	0%	0/5	20%	1/5	0%	0/5	40%	6/15	7%	1/15
SR.A.2.d (3D Figure)	33%	2/6	17%	1/6	40%	2/5	40%	2/5	60%	3/5	20%	2/5	44%	7/16	25%	4/16
SR.A.2.e (Fold)	50%	3/6	20%	1/5	40%	2/5	0%	0/4	40%	2/5	0%	0/4	44%	7/16	8%	1/13
SR.A.2.e (Fold & Punch)	50%	3/6	25%	1/4	0%	0/5	20%	1/5	60%	3/5	40%	2/5	38%	6/16	29%	4/14
SR.A.3.a (2D)	17%	1/6	0%	0/6	0%	0/5	0%	0/5	0%	0/5	0%	0/5	6%	1/16	0%	0/16
SR.A.3.a (3D)	0%	0/6	0%	0/6	0%	0/5	0%	0/5	0%	0/4	0%	0/5	0%	0/16	0%	0/15
SR.A.3.b (2D)	67%	4/6	0%	0/6	100%	5/5	0%	0/5	40%	2/5	0%	0/5	69%	11/16	0%	0/16
SR.A.3.b (3D)	0%	0/6	0%	0/6	20%	1/5	0%	0/5	0%	0/5	0%	0/5	6%	1/16	0%	0/16
SR.A.3.c (2D)	0%	0/6	0%	0/6	0%	0/5	0%	0/5	20%	1/5	20%	1/5	6%	1/16	6%	1/16
SR.A.3.c (3D)	0%	0/6	33%	2/6	0%	0/5	20%	1/5	20%	1/5	0%	0/5	6%	1/16	15%	2/14
SR.A.3.d	33%	2/6	17%	1/6	20%	1/5	0%	0/5	0%	0/5	0%	0/5	19%	3/16	6%	1/16
SR.A.3.e	0%	0/5	20%	1/5	0%	0/5	0%	0/5	20%	1/5	0%	0/5	7%	1/15	7%	1/15
SR.A.3.f	40%	2/5	20%	1/5	40%	2/5	0%	0/2	60%	3/5	0%	0/3	47%	7/15	10%	1/10
SR.A.3.g	0%	0/5	40%	2/5	0%	0/4	0%	0/4	0%	0/5	20%	1/5	0%	0/14	21%	3/14
SR.A.3.a (2D)	17%	1/6	0%	0/6	0%	0/5	0%	0/5	0%	0/5	0%	0/5	6%	1/16	0%	0/16
SR.A.3.a (3D)	0%	0/6	0%	0/6	0%	0/5	0%	0/5	0%	0/4	0%	0/5	0%	0/16	0%	0/15

Within: Repeat by Grade

Within	Kindergarten				Grade 1				Grade 2				All Grades K-2			
Subcomponent	Need to Repeat CQ (%) [KG]	KG All n repeat CQ	Need to Repeat RQ (%) [KG]	KG All n repeat RQ	Need to Repeat CQ (%) [1st]	G1 All n repeat RQ	Need to Repeat RQ (%) [1st]	G1 All n repeat RQ	Need to Repeat CQ (%) [2nd]	G2 All n repeat RQ	Need to Repeat RQ (%) [2nd]	G2 All n repeat RQ	Need to Repeat CQ (%) [ALL]	All n repeat RQ	Need to Repeat RQ (%) [ALL]	Total n repeat RQ
SR.A.1.a	33%	2/6	40%	2/5	20%	1/5	20%	1/5	0%	0/5	0%	0/5	19%	3/16	20%	3/15
SR.A.1.b (2D)	33%	2/6	17%	1/6	0%	0/5	60%	3/5	33%	1/3	20%	1/5	21%	3/14	31%	5/16
SR.A.1.c/d (2D)	17%	1/6	17%	1/6	0%	0/5	0%	0/4	0%	0/5	0%	0/5	6%	1/16	7%	1/15
SR.A.1.b.(3D)	17%	1/6	0%	0/6	0%	0/5	0%	0/5	20%	1/5	0%	0/5	13%	2/16	0%	0/16
SR.A.1.c/d.(3D)	17%	1/6	17%	1/6	20%	1/5	25%	1/4	0%	0/5	20%	1/5	13%	2/16	20%	3/15
SR.A.2.a/b	33%	2/6	17%	1/6	0%	0/5	40%	2/5	0%	0/5	20%	1/5	13%	2/16	25%	4/16
SR.A.2.c (2D Shape)	20%	1/5	0%	0/5	20%	1/5	40%	2/5	20%	1/5	0%	0/5	20%	3/15	13%	2/15
SR.A.2.d (2D Shape)	20%	1/5	40%	2/5	0%	0/5	40%	2/5	0%	0/5	20%	1/5	7%	1/15	33%	5/15
SR.A.2.d (3D Figure)	17%	1/6	33%	2/6	0%	0/5	25%	1/4	0%	0/5	20%	1/5	6%	1/16	27%	4/15
SR.A.2.e (Fold)	17%	1/6	20%	1/5	20%	1/5	25%	1/4	20%	1/5	0%	0/4	19%	3/16	15%	2/13
SR.A.2.e (Fold & Punch)	33%	2/6	25%	1/4	0%	0/5	20%	1/5	20%	1/5	20%	1/5	19%	3/16	21%	3/14
SR.A.3.a (2D)	0%	0/6	0%	0/6	0%	0/5	0%	0/5	0%	0/5	0%	0/5	0%	0/16	0%	0/16
SR.A.3.a (3D)	0%	0/6	0%	0/6	0%	0/5	0%	0/5	0%	0/5	0%	0/4	0%	0/16	0%	0/15
SR.A.3.b (2D)	0%	0/6	17%	1/6	0%	0/5	0%	0/5	20%	1/5	0%	0/5	6%	1/16	6%	1/16
SR.A.3.b (3D)	0%	0/6	0%	0/6	0%	0/5	0%	0/5	20%	1/5	0%	0/5	6%	1/16	0%	0/16
SR.A.3.c (2D)	0%	0/6	0%	0/6	0%	0/5	0%	0/5	0%	0/5	0%	0/5	0%	0/16	0%	0/16
SR.A.3.c (3D)	33%	2/6	0%	0/4	0%	0/5	0%	0/5	40%	2/5	0%	0/5	25%	4/16	0%	0/14
SR.A.3.d	0%	0/6	0%	0/6	0%	0/5	0%	0/5	20%	1/5	0%	0/5	6%	1/16	0%	0/16
SR.A.3.e	40%	2/5	20%	1/5	0%	0/5	0%	0/5	0%	0/5	0%	0/5	13%	2/15	7%	1/15
SR.A.3.f	20%	1/5	0%	0/5	0%	0/5	0%	0/2	0%	0/5	0%	0/3	7%	1/15	0%	0/10
SR.A.3.g	0%	0/5	0%	0/5	0%	0/4	0%	0/4	20%	1/5	0%	0/5	7%	1/14	0%	0/14

Within: Comfort with Material

Within	Kindergarten		Grade 1		Grade 2		All Grades K-2	
Subcomponent	Comfort With Material (%) [KG]	KG All n comfort with materials	Comfort With Material (%) [1st]	G1 All n comfort with materials	Comfort With Material (%) [2nd]	G2 All n comfort with materials	Comfort With Material (%) [ALL]	Total n comfort with materials
SR.A.1.a	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.1.b (2D)	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.1.c/d (2D)	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.1.b.(3D)	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.1.c/d.(3D)	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.2.a/b	100%	6/6	100%	5/5	100%	5/5	100%	16/16
aSR.A.2.c (2D Shape)	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.2.d (2D Shape)	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.2.d (3D Figure)	100%	5/5	100%	5/5	100%	5/5	100%	15/15
SR.A.2.e (Fold)	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.2.e (Fold & Punch)	100%	6/6	100%	4/4	100%	5/5	100%	15/15
SR.A.3.a (2D)	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.3.a (3D)	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.3.b (2D)	100%	6/6	80%	4/5	100%	5/5	94%	16/16
SR.A.3.b (3D)	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.3.c (2D)	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.3.c (3D)	100%	6/6	100%	5/5	80%	4/5	94%	16/16
SR.A.3.d	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.A.3.e	100%	5/5	100%	5/5	100%	5/5	100%	15/15
SR.A.3.f	100%	5/5	100%	5/5	100%	5/5	100%	15/15
SR.A.3.g	100%	5/5	100%	5/5	100%	5/5	100%	15/15

Within: Comfort with Task

Within	Kindergarten		Grade 1		Grade 2		All Grades K-2	
Subcomponent	Average Score of Comfort with Task (3) [KG]	KG All n comfortable level	Average Score of Comfort with Task (3) [1st]	G1 All n comfortable level	Average Score of Comfort with Task (3) [2nd]	G2 All n comfortable level	Average Score of Comfort with Task (3) [ALL]	Total n comfort level
SR.A.1.a	2.67	6	2.20	5	2.60	5	2.50	16
SR.A.1.b (2D)	2.67	6	2.40	5	2.60	5	2.56	16
SR.A.1.c/d (2D)	2.67	6	2.60	5	2.40	5	2.56	16
SR.A.1.b.(3D)	2.67	6	2.80	5	2.80	5	2.75	16
SR.A.1.c/d.(3D)	2.83	6	2.40	5	3.00	5	2.75	16
SR.A.2.a/b	2.83	6	2.60	5	3.00	5	2.81	16
aSR.A.2.c (2D Shape)	2.83	6	2.80	5	3.00	5	2.88	16
SR.A.2.d (2D Shape)	2.80	6	2.80	5	3.00	5	2.80	16
SR.A.2.d (3D Figure)	2.83	5	2.60	5	3.00	5	2.81	15
SR.A.2.e (Fold)	2.83	6	2.60	5	3.00	5	2.81	16
SR.A.2.e (Fold & Punch)	2.83	6	2.75	4	3.00	5	2.87	15
SR.A.3.a (2D)	3.00	6	2.40	5	3.00	5	2.81	16
SR.A.3.a (3D)	3.00	6	2.60	5	3.00	5	2.88	16
SR.A.3.b (2D)	3.00	6	2.40	5	2.80	5	2.75	16
SR.A.3.b (3D)	3.00	6	2.80	5	2.40	5	2.75	16
SR.A.3.c (2D)	2.83	6	2.60	5	3.00	5	2.81	16
SR.A.3.c (3D)	3.00	6	3.00	5	3.00	5	2.88	16
SR.A.3.d	3.00	6	3.00	5	3.00	5	3.00	16
SR.A.3.e	2.80	5	2.80	5	2.80	5	2.80	15
SR.A.3.f	2.80	5	2.60	5	2.80	5	2.73	15
SR.A.3.g	2.80	5	2.80	5	3.00	5	2.87	15
Average	2.84		2.65		2.87		2.78	

Between Objects TLG

Between: Reword by Grade

Between	Kindergarten				Grade 1				Grade 2				All Grades K-2			
Subcompon-ent	Need to Reword CQ (%) [KG]	KG All n reword	Need to Reword RQ (%) [KG]	KG All n reword	Need to Reword CQ (%) [1st]	G1 All n reword	Need to Reword RQ (%) [1st]	G1 All n reword	Need to Reword CQ (%) [2nd]	G2 All n reword	Need to Reword RQ (%) [2nd]	G2 All n reword	Need to Reword CQ (%) [ALL]	All n reword	Need to Reword RQ (%) [ALL]	Total n reword
SR.B.5.a	33%	2/6	67%	4/6	20%	1/5	20%	1/5	40%	2/5	40%	2/5	31%	5/16	44%	7/16
SR.B.5.b	17%	1/6	0%	0/6	0%	0/5	0%	0/5	0%	0/5	20%	1/5	6%	1/16	6%	1/16
SR.B.5.c	67%	4/6	33%	2/6	40%	2/5	0%	0/5	60%	3/5	25%	1/4	56%	9/16	20%	3/15
SR.B.6.a	17%	1/6	17%	1/6	20%	1/5	20%	1/5	0%	0/5	20%	1/5	13%	2/16	19%	3/16
SR.B.6.b	17%	1/6	17%	1/6	20%	1/5	20%	1/5	20%	1/5	20%	1/5	19%	3/16	19%	3/16
SR.B.6.c	17%	1/6	33%	2/6	40%	2/5	40%	2/5	0%	0/5	20%	1/5	19%	3/16	31%	5/16
SR.B.6.d	67%	4/6	50%	3/6	40%	2/5	40%	2/5	40%	2/5	0%	0/5	50%	8/16	31%	5/16
SR.B.6.e	33%	2/6	33%	2/6	20%	1/5	20%	1/5	20%	1/5	0%	0/5	25%	4/16	19%	3/16
SR.B.6.f	17%	1/6	33%	2/6	0%	0/5	20%	1/5	0%	0/5	0%	0/4	6%	1/16	20%	3/15
SR.B.6.g	0%	0/6	0%	0/6	0%	0/5	20%	1/5	0%	0/5	40%	2/5	0%	0/16	19%	3/16
SR.B.6.h	17%	1/6	83%	5/6	0%	0/5	40%	2/5	0%	0/5	60%	3/5	6%	1/16	63%	10/16
SR.B.7.a	67%	4/6	33%	2/6	0%	0/5	20%	1/5	0%	0/5	0%	0/5	25%	4/16	19%	3/16
SR.B.7.b	0%	0/6	17%	1/6	0%	0/5	40%	2/5	0%	0/5	40%	2/5	0%	0/16	31%	5/16
SR.B.7.c	33%	2/6	0%	0/6	40%	2/5	40%	2/5	40%	2/5	20%	1/5	38%	6/16	19%	3/16
SR.B.7.d	17%	1/6	25%	1/4	0%	0/5	0%	0/5	0%	0/5	20%	1/5	6%	1/16	14%	2/14
SR.B.7.e (Task 1)	17%	1/6	80%	4/5	0%	0/5	0%	0/5	20%	1/5	40%	2/5	13%	2/16	40%	6/15
SR.B.7.e (Task 2)	0%	0/4	50%	2/4	0%	0/5	0%	0/1	0%	0/2	0%	0/1	0%	0/7	33%	2/6

Between: Repeat by Grade

Between	Kindergarten				Grade 1				Grade 2				All Grades K-2			
Subcompon-ent	Need to Repeat CQ (%) [KG]	KG All n repeat	Need to Repeat RQ (%) [KG]	KG All n repeat	Need to Repeat CQ (%) [1st]	G1 All n repeat	Need to Repeat RQ (%) [1st]	G1 All n repeat	Need to Repeat CQ (%) [2nd]	G2 All n repeat	Need to Repeat RQ (%) [2nd]	G2 All n repeat	Need to Repeat CQ (%) [ALL]	All n repeat	Need to Repeat RQ (%) [ALL]	Total n repeat
SR.B.5.a	17%	1/6	0%	0/6	0%	0/5	20%	1/5	0%	0/5	0%	0/5	6%	1/16	6%	1/16
SR.B.5.b	33%	2/6	0%	0/6	20%	1/5	0%	0/5	0%	0/5	20%	1/5	19%	3/16	6%	1/16
SR.B.5.c	0%	0/6	17%	1/6	20%	1/5	0%	0/5	20%	1/5	0%	0/4	13%	3/16	7%	1/15
SR.B.6.a	17%	1/6	0%	0/6	20%	1/5	0%	0/5	20%	1/5	20%	1/5	19%	3/16	6%	1/16
SR.B.6.b	17%	1/6	0%	0/6	20%	1/5	20%	1/5	20%	1/5	0%	0/5	19%	3/16	6%	1/16
SR.B.6.c	0%	0/6	0%	0/6	0%	0/5	20%	1/5	0%	0/5	0%	0/5	0%	0/16	6%	1/16
SR.B.6.d	0%	0/6	0%	0/6	0%	0/5	20%	1/5	0%	0/5	0%	0/5	0%	0/16	6%	1/16
SR.B.6.e	33%	2/6	0%	0/6	40%	2/5	20%	1/5	0%	0/5	0%	0/5	25%	4/16	6%	1/16
SR.B.6.f	17%	1/6	0%	0/6	0%	0/5	0%	0/5	0%	0/5	0%	0/4	6%	1/16	0%	0/15
SR.B.6.g	33%	2/6	0%	0/6	0%	0/5	0%	0/5	0%	0/4	0%	0/5	13%	2/15	0%	0/16
SR.B.6.h	33%	2/6	17%	1/6	0%	0/5	0%	0/5	0%	0/5	0%	0/5	13%	2/16	6%	1/16
SR.B.7.a	17%	1/6	0%	0/6	0%	0/5	20%	1/5	0%	0/5	0%	0/5	6%	1/16	6%	1/16
SR.B.7.b	17%	1/6	33%	2/6	0%	0/5	0%	0/5	0%	0/5	0%	0/5	6%	1/16	13%	2/16
SR.B.7.c	33%	2/6	17%	1/6	20%	1/5	0%	0/5	20%	1/5	0%	0/5	25%	4/16	6%	1/16
SR.B.7.d	20%	1/5	25%	1/4	0%	0/5	0%	0/5	0%	0/5	20%	1/5	7%	1/15	14%	2/14
SR.B.7.e (Task 1)	33%	2/6	0%	0/5	0%	0/5	0%	0/5	20%	1/5	0%	0/5	19%	3/16	0%	0/15
SR.B.7.e (Task 2)	0%	0/4	0%	0/4	0%	0/1	0%	0/1	0%	0/2	0%	0/1	0%	0/7	0%	0/6

Between: Comfort with Manipulatives

Between	Kindergarten		Grade 1		Grade 2		All Grades K-2	
Subcomponent	KG All Comf. With Manipulatives (%)	KG All n comfortable manipulatives	G1 All Comf. With Manipulatives (%)	G1 All n comfortable manipulatives	G2 All Comf. With Manipulatives (%)	G2 All n comfortable manipulatives	Average comfort level	Total n comfort level
	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.5.b	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.5.c	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.6.a	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.6.b	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.6.c	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.6.d	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.6.e	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.6.f	100%	5/5	100%	5/5	100%	5/5	100%	15/15
SR.B.6.g	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.6.h	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.7.a	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.7.b	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.7.c	100%	6/6	100%	5/5	100%	5/5	100%	16/16
SR.B.7.d	100%	5/5	100%	5/5	100%	5/5	100%	15/15
SR.B.7.e (Task 1)	83%	5/6	100%	5/5	100%	5/5	94%	15/16
SR.B.7.e (Task 2)	100%	4/4	100%	1/1	100%	2/2	100%	7/7

Between: Comfort with Task

Between	Kindergarten		Grade 1		Grade 2		All Grades K-2	
Subcomponent	Average Score of Comfort with Task (3) [KG]	KG All n comfortable manipulatives	Average Score of Comfort with Task (3) [1st]	G1 All n comfortable manipulatives	Average Score of Comfort with Task (3) [2nd]	G2 All n comfortable manipulatives	Average Score of Comfort with Task (3) [ALL]	Total n comfort level
SR.B.5.a	2.67	6	2.60	5	2.40	5	2.56	16
SR.B.5.b	2.83	6	2.80	5	2.60	5	2.75	16
SR.B.5.c	2.83	6	2.40	5	2.60	5	2.63	16
SR.B.6.a	2.83	6	2.80	5	2.80	5	2.81	16
SR.B.6.b	3.00	6	2.80	5	2.80		2.88	16
SR.B.6.c	2.83	6	2.80	5	2.80	5	2.81	16
SR.B.6.d	2.83	6	2.80	5	2.80	5	2.81	16
SR.B.6.e	3.00	6	2.80	5	2.60	5	2.81	16
SR.B.6.f	2.83	6	3.00	5	3.00	5	2.94	16
SR.B.6.g	2.83	6	2.60	5	3.00	5	2.81	16
SR.B.6.h	2.67	6	2.80	5	3.00	5	2.81	16
SR.B.7.a	3.00	6	2.80	5	3.00	5	2.94	16
SR.B.7.b	2.67	6	3.00	5	3.00	5	2.88	16
SR.B.7.c	2.67	6	2.60	5	3.00	5	2.75	16
SR.B.7.d	2.40	5	2.60	5	3.00	5	2.67	15
SR.B.7.e (Task 1)	2.50	5	2.60	5	2.80	5	2.63	16
SR.B.7.e (Task 2)	3.00	4	3.00	1	3.00	2	3.00	7
Avg.	2.79		2.75		2.84		2.79	