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Spatial Reasoning: Cognitive Interview Protocol Development

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Abstract

The purpose of the current report is to describe the Spatial Reasoning (SR) cognitive interview development process for the Measuring Early Mathematics Reasoning and Skills (MMaRS) project. We developed the cognitive interviews to reflect the skills outlined in the SR learning progression. Along with expert reviews and a teacher survey, the data collected with the cognitive interviews will assist in the empirical recovery of the SR learning progression.

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Spatial Reasoning: Cognitive Interview Protocol Development

Introduction

The Spatial Reasoning (SR) construct consists of two components: spatial orientation and spatial visualization (Bishop, 1980; Burnett & Lane, 1980; Clements & Battista, 1992; Connor & Serbin, 1980; Eliot & Smith, 1983; NRC, 2009; Pellegrino et al., 1984; Sarama & Clements, 2009; Tartre, 1990). Spatial orientation involves identifying one's own position and how that position is related to the world, including taking alternate perspectives. Spatial visualization is the ability to mentally create and transform two-dimensional and three-dimensional figures (McGee, 1979), including mental rotation, reflection, and transformation of objects and shapes (Clements, 2004; Linn & Petersen, 1985). Within the Measuring Early Mathematical Reasoning Skills (MMaRS) project, SR was conceptualized as these two targeted learning goals, Reasoning Spatially Between and Within Objects. Between Objects requires use of spatial orientation and Within Objects requires spatial visualization.

The SR Learning Progression (LP) was developed around both SR targeted learning goals to provide a foundation to a K-2 assessment instrument focused on the overall SR construct. Based on each targeted learning goal, test items were developed spanning grade levels that included elements that vary. For more information about the development of the SR LP, please see the SR Learning Progression Development technical report (Tech. Rep. No. 20-06).

The purpose of these cognitive interviews was to work in conjunction with the SR Teacher Survey technical report (Tech. Rep. No. 20-10) as the MMaRS team began the process of confirming and disconfirming the ordering and conceptualization of the LP. We developed protocols for try-outs and cognitive interviews hosted to gather empirical evidence on student strategies and problem-solving capacity within the hypothesized SR content and constructs. The purpose of this technical report is to describe the development of the protocols.

Research Questions

The primary purpose of the Spatial Reasoning cognitive interviews is to provide validity evidence to empirically recover the MMaRS Spatial Reasoning learning progressions. Specifically, four focal areas will be addressed including the conceptualization of content, ordering, developmental appropriateness, and interconnectedness of the learning progression.

Overarching Research Question

To what extent are the Spatial Reasoning Learning Progressions valid learning progressions?

The overarching research question encompasses the four focal areas of this study: conceptualization of content, ordering, developmentally appropriateness, and interconnectedness.

Boundaries of a skill statement will be addressed within the developmentally appropriateness focal area and developmentally appropriateness is distinguished from conceptualization of content. The following sections include an overview of each of the four focal areas.

Conceptualization of Content

Each essentialized skill statement has a specified grade band and may include specific strategies associated within the skill. As such, the focal area of conceptualization of content, which focuses on the framing or wording of the skill, is distinct from developmental appropriateness which focuses on the boundaries of a skill statement. To further address boundaries within core concepts, elements that vary were included based on child development and standards-based expectations for student application of knowledge.

Ordering

Ordering focuses specifically on the vertical order of skill statements within a core concept. Horizontal boundaries were addressed within developmentally appropriateness to clearly delineate the differences in the types of ordering.

Developmentally Appropriateness

Grade level designations and boundaries of skill statements are addressed in the focal area of developmentally appropriateness. The process of looking at vertical ordering was a process that naturally occurred after addressing the wording and the boundaries of each of the skill statements. Developmental appropriateness was further informed through grade level standards, and the timing at which concepts and vocabulary are introduced.

Interconnectedness

The interconnectedness focal area addresses how students may incorporate other skills within the learning progression for any given skill. Evidence of interconnectedness supports the inclusion of skills in each of the learning progressions.

The qualitative data from the SR cognitive interviews will be analyzed to provide confirming or disconfirming evidence for each of the focal areas described above.

Protocol Development

The cognitive interview protocols were designed to investigate students' knowledge and skills regarding each of the subcomponents within the Spatial Reasoning Learning Progressions. The protocols included tasks for each of the skill statements on the LPs and each progressed from least complex skill to most complex skill.

The team followed an iterative development cycle. Each protocol presented items with content and reasoning questions for each skill statement in the learning progression. To ensure that items were eliciting intended conceptual and procedural thinking, the development team conducted a series of item development cycles, including two rounds of interview try-outs, to finalize the

interview protocol. This iterative process allowed the team to synthesize trends in student responses and perceived misconceptions, and to refine and replace items to better elicit the intended spatial reasoning constructs.

SR protocols were similar in structure to the NRR protocols. One distinct difference, however, was the designation of task complexity. In NRR, task complexity was associated with the number range for each item; for SR protocols, task complexity related to elements that vary.

Development Team

Initial development occurred in spring and summer, 2019 by a researcher and research project manager. The protocols were developed delineated by Reasoning Spatially Within Objects and Between Objects. Portions of the Within Objects protocol were administered to four children as a tryout in May, 2019, with each Essentialized Skill Statement task administered to a minimum of two children.

Based on tryout round one data for Within Objects protocol, a research assistant refined the protocols in Fall, 2019. Steps in September, 2019 included adding additional context to item and scenario setup, revising and extending content and reasoning questions, and developing during and after interview scoring procedures. The Co-PI and research project manager supported the refinement through regular reviews and group meetings to talk through challenges and provide alternate perspectives and literature for review. In October, the revised protocols were sent to the PI, and a final round of revisions was completed based on feedback before round two tryouts in November, 2019. and initial development tasks on the Between

Learning Progression Overview

Learning Progressions were developed through literature review, synthesis, and internal and external reviews of content and ordering. See the Learning Progression Technical Report for full details on the development and pre-interview refinement of the LPs (Tech. Rep. No. 20-06).

LPs were based on Reasoning Spatially Within Objects and Reasoning Spatially Between Objects, which can be viewed through a lens of intrinsic and extrinsic perspective of the spatial reasoning. Core concepts and skills that aligned with a single object, whether static or dynamic, and the changes it could undergo intrinsically were included in the Within Objects progression. Core concepts and skills that drew on perception taking and movement between more than one object were included in the Between Objects progression . It is important to note that Within Objects is numbered A.1. to A.3. and Between Objects is B.5. to B.7. (see Figure 1 for sample LP core concept). Through iterations of the LPs, a fourth core concept of Within Objects was absorbed into the first three as the concepts and their definitions were detailed and defined, but numbering for Between Objects was not changed.

Figure 1

Core Concept Exemplar with Subcomponent Skill Statements

7. Perspective Taking									
Code	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
SR.B.7.a	Recognize the view from one’s own perspective.								
SR.B.7.b	Understand that changes in perspective changes the view .								
SR.B.7.c		Describe relative spatial positions of objects from different perspectives (e.g., “the chair would be closest to me if I stood over there”).							
SR.B.7.d		Recognize views from different perspectives (e.g., identifies what photo could be taken from a specific viewpoint of a concrete or pictorial representation of a three-dimensional space or object).							
SR.B.7.e		Construct a three-dimensional object or space given at least two images of top, front, or side views.							

Note. Core concept 7 with subcomponent statements from the Reasoning Spatially Between Objects targeted learning goal, with developmental appropriateness bands outlines.

Protocol Alignment and Content Questions

For each core concept, subcomponent skill statements detailed the skills required in the overall construct. The development team worked carefully through the LPs to create items that assessed the designated skills and fit within the overall context of the protocol as it was developed. Reasoning Spatially Within Objects contained items that relied heavily on two- and three-dimensional shapes and the construction or deconstruction of figures with blocks. In this protocol, the items could be developed without specific context as each skill was intrinsic to the item and its object. Content questions were developed specific to identifying, transforming, and composing or decomposing the shapes as required by the skill.

For Reasoning Spatially Between Objects, more contextualization was needed to develop questions that included perspective taking and spatial language. The three included Core Concepts were constructed around a farm and a classroom diorama, so that questions could be situated within a story. By developing this context, students were able to tell stories about the movement of animals or placement of objects on the farm that provided answers to content questions. The content questions required students to take perspective, scale, and use positional language either at the farm or on maps and dioramas as provided.

Reasoning Questions

Reasoning questions were developed through iterations of the protocol using anticipated responses or strategies and informed by the NRR reconciliation process. The intent was to capture students’ conceptions of anticipated strategies and uncover misconceptions or unanticipated strategies that would serve to support or refute the content and ordering of the LPs.

By carefully crafting reasoning probes with a combination of positive and negative questions, a variety of questions were posed to begin uncovering how the children were thinking about each of the scenarios presented. Children were given opportunities to explain either why they selected the answer that they did and why they did not select a different answer. Interviewers were to never confirm or disconfirm correctness when asking these questions to avoid leading children.

Elements that Vary

To begin understanding the floor and ceiling effects for skills within the learning progression, elements that vary were introduced based on grade and age-level designations. The elements that vary in the SR protocols included shapes, embedded figures, two- and three-dimensional composite figures, translations, rotations, reflections, cross-sections, positional language and route complexity (See Figure 2).

The grade and age-level bands were hypothesized within the pre-cognitive interview content and ordering of the learning progressions. Based on CI outcomes and reconciliation, the elements that vary will be updated to best support the learning progression at that time.

Figure 2

Elements that Vary from the Reasoning Spatially Within Objects Protocol

A. Reasoning Spatially Within Objects

Elements that Vary	Kindergarten			Grade 1			Grade 2		
	F	B	T	F	B	T	F	B	T
Shapes (all can be regular or irregular)	Regular or irregular circles, squares, triangles, rectangles		Regular or irregular hexagons, rhombus, cubes, cones, cylinders, spheres, pyramids, prisms, trapezoids				Regular or irregular quadrilaterals		
Embedded figure			The larger figure is a whole shapes (e.g., rectangle) without any other line segments crossing through the shape.		The larger figure is a Figure made up of line segments (e.g., triangle) with the possibility of other line segments crossing through the figure.		The larger figure is a line segment group (e.g., lines formed as an "L") with the possibility of other line segments crossing through the figure.		
Two-dimensional composite figure	Outline of all shapes are shown.		Shapes may share one side but not all outlines are shown.			Shapes may share more than one side but not all outlines are shown.			
Three-dimensional composite figure	All shapes can be seen.				There may be internal spaces or blocks that can't be seen but that are necessary to the structural integrity.				

Tryouts

Four students in grades K-2 participated in spring 2019 first-round tryouts on the Within Objects protocol only, six students in grades K-2 participated in fall 2019 second-round tryouts, and two students in grades 1-2 participated fall 2019 third-round tryouts. Each student engaged one-on-one with an interviewer through one of two protocols. (give overall demographics – male/female grade breakout). One SR subcomponent was assigned to each student. Each of the two interview protocol developers were slated to implement the interviews during data collection and conducted two tryout interviews. Then, the tryout interviews were used to train the observers in

their specific roles for data collection across each of the three NRR subcomponents. Since the interviewers were also the developers of the cognitive interview protocols, the tryout interviews served as training for the interviewers.

Next Steps

We made final changes to the cognitive interview protocols based upon the tryouts. Next, we implement these protocols with a larger sample of students. In conjunction with expert reviews and teacher survey results, we will use these data to empirically recover the SR learning progression. Details about the cognitive interview data collection process and data analyses can be found in the Spatial Reasoning Cognitive Interview Administration technical report (Tech. Rep. No. 20-23).

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