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Research in Mathematics Education

# Imagination Station (Istation):

# Universal Screener Instrument Development for Grade 3

**Technical Report 12-02**

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# Universal Screener Instrument Development for Grade 3


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## Abstract

In this technical report, we describe the development of the Grade 3 formative assessment item bank for the Imagination Station (Istation). The formative assessment item bank will be used to deliver a computerized adaptive universal screening assessment to support teachers' instructional decision-making. State and national standards of mathematics skills and knowledge for Grade 3 inform the construct underlying the items. We include a description of the process used to identify and sample the mathematics content and levels of cognitive complexity assessed in the item bank. Next, we describe the item writing procedures. Finally, we describe how the external item review process and outcomes impact content-related evidence for validity.

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# Imagination Station (Istation): Universal Screener Instrument Development for Grade 3

## Introduction

The purpose of the Grade 3 formative assessment item bank for the Imagination Station (Istation) is to support teachers' instructional decision-making. The formative assessment item bank is a computerized adaptive universal screening assessment system to monitor student progress with fundamental mathematics skills and grade level standards. By administering this assessment system, teachers and administrators can use the results to answer two questions: (1) are students at risk of failure in Grade 3 mathematics, and (2) what is the degree of intensity of instructional support students need to be successful in Grade 3 mathematics? Multiple administrations of the universal screener (i.e., fall, winter, and early spring each year) provide teachers with meaningful information about student progress to support instructional decision-making over the course of Grade 3. The universal screener is designed for administration to all students receiving grade-level instruction.

The purpose of this technical report is to describe the development of the formative assessment item bank. This description includes (1) the process used to identify and sample the mathematics content assessed in the item bank, (2) the item writing process, and (3) the external review process and results. The test development steps used to create the formative assessment item bank represent best practices in test development and the Test Standards published by the American Educational Research Association (AERA), American Psychological Association (APA), and National Council on Measurement in Education (NCME) (1999).

## Construct Definition

The assessment construct consists of (1) mathematics content and (2) level of cognitive engagement. The content of the Grade 3 formative assessment item bank is based on the Curriculum Focal Points (National Council of Teachers of Mathematics [NCTM], 2006), mathematics content standards published by the Common Core Standards Initiative, and state standards from Texas, Florida, New York, California, and Virginia. See Appendix A for the state content standards. To create the assessment construct, we aligned the Common Core standards and state standards to the Curriculum Focal Points (CFP). We created a fourth CFP to include two standards that were assessed across the states but was not represented in the NCTM focal points: representing and interpreting data; and geometry and measurement (e.g., currency, temperature, and time). See Appendix B for an abbreviated description of the assessed content.

The cognitive engagement dimension of the construct refers to the level of cognitive processing at which students are expected to engage an assessment item. The formative assessment item bank uses the taxonomy of cognitive engagement in mathematics published by Kilpatrick, Swafford, and Findell (2001) for the National Research Council. The taxonomy consists of five interdependent strands that promote mathematical proficiency: (1) conceptual understanding, (2) procedural fluency, (3) strategic competence, (4) adaptive reasoning, and (5) productive disposition. The formative assessment item bank assesses student understanding of the content at varying levels of cognitive engagement. A brief description of each level follows:

1. Conceptual understanding pertains to the functional grasp of mathematics that a student applies to concepts, operations, and relations. It involves being able to logically organize one's knowledge to integrate and understand concepts as part of a coherent whole.

2. Procedural fluency pertains to students' ability to accurately and appropriately carry out skills, including being able to select efficient and flexible approaches.

3. Strategic competence involves student's ability to formulate a problem in mathematical terms, to represent it strategically (verbally, symbolically, graphically, or numerically), as well as to solve it effectively. It is similar to problem solving and problem formation.

4. Adaptive reasoning involves the student's capacity to think logically about a problem, which requires reflecting on various approaches to solve a problem and deductively selecting an approach. Students who are able to do this are also able to rationalize and justify their strategy.

5. Productive disposition refers to a student's overall ability to perceive mathematics as worthwhile and to maintain a personal belief in one's own efficacy in solving problems.

The formative assessment item bank incorporates four of the five strands. Productive disposition is not assessed.

Each CFP was assessed at the four levels of cognitive engagement. Conceptual understanding and procedural fluency were oversampled to accurately reflect the relative emphasis in the state standards. Easy, medium, and difficult items were written for each CFP across the four levels of cognitive engagement. The content sampling matrix is presented in Figure 1.

# Item Writing

## Item Specifications

Approximately 400 items were written for Grade 3. Multiple-choice items were created for efficiency in the computer delivery system. Each item had three distractors and one correct answer. Items were scored dichotomously as either right or wrong. The distractors represent plausible misconceptions or errors in computation, procedure, conceptual understanding, and strategy.

The item stem included text and/or graphics. The language used in all text was intentionally constrained to the 3rd grade level; however, readability statistics were not calculated for each item. Whenever possible, plain language and simple, straightforward statements were incorporated into the items. Graphics were used in instances where they explained the problem, provided a visual clue to clarify the context, or were integral to the stem or answer choices. Irrelevant graphics were not included.

The assessment items were written according to the principles of universal design for assessment (See Ketterlin-Geller, 2005; 2008) and are amendable to accommodations. The formative assessment system includes a read aloud feature to support item readability. This ensures that mathematics ability is tested, rather than students' reading ability.

The computerized-adaptive test can be administered individually or in a group in an untimed setting.

## Item Writers

Seven item writers contributed items to the Grade 3 formative assessment item bank.

**Item Writer 1.** Item Writer 1 holds a Bachelors of Arts in Psychology and Masters degrees in Counseling Psychology, Special Education, and School Psychology. She received her Ph.D. from the University of Oregon in Educational Leadership where she focused on assessment and solid assessment development procedures as well as psychometric procedures to evaluate assessments once developed. After graduating, she worked for a nonprofit organization where she assisted in the design, development, and evaluation of education programs and improvement initiatives. She later worked as a school psychologist where she conducted comprehensive psycho-educational evaluations to determine student eligibility for special services and to inform interventions. She currently works as a special education program specialist for a school district and periodically serves as an assessment consultant on projects.

**Item Writer 2.** Item Writer 2 holds a Bachelors of Science and a Masters degree in Special Education. He has been a research assistant, project coordinator and independent contractor for federally funded grants and state contracts since 2001. He assisted in the creation of a web-delivered math assessment researching effectiveness of accommodations. He was also involved in developing an alternate assessment for elementary, middle, and high school students with significant cognitive disabilities between 2002 and 2009. He also helped to write and create items for mathematics screening tests, as well as to develop accommodated versions of items. He has been a part of several research teams conducting multi-state research projects examining comparability of performance on alternate assessments.

**Item Writer 3.** Item Writer 3 holds a Bachelor of Arts degree in Elementary Education and a Masters of Education in Curriculum and Instruction. She is also a Texas state certified Master Reading Teacher. Currently, she is a Ph.D. candidate at Southern Methodist University where she has worked on several technology-based assessment and professional development grants. She assisted in developing items for statewide universal screening and diagnostic assessments focusing on algebra-readiness in middle school students. Before entering the doctoral program, Item Writer 3 spent over 18 years as a classroom teacher and administrator. As the Director of Curriculum at a PK-8 school of over 800 children she developed academic and professional development programs, mentored teachers, administered formative and summative teacher evaluations, and supervised the administration of assessments in reading and mathematics.

**Item Writer 4.** Item Writer 4 earned a Bachelor of Science in Mathematics and a Master of Science in Mathematics Education from Oregon State University. She taught mathematics for six years at the middle, high school, and community college level. In addition to teaching, she currently works as a mathematics coach in her school district. In this position, she focuses on improving instruction across the district by developing curriculum that is aligned to state mathematics standards. Her interest in assessments led her to become an item-writer for mathematics assessments.

**Item Writer 5.** Item Writer 5 graduated from Texas Christian University with a Bachelor of Science degree in Mathematics Education. She taught high school Geometry and Pre-Calculus for three years. While teaching, she earned her M.Ed. degree from The University of Texas at Arlington in Mathematics Curriculum and Instruction. She also tutors students in subjects ranging from eighth grade mathematics to Pre-Calculus.

**Item Writer 6.** Item Writer 6 received her B.A. in Mathematics from the University of Texas at Austin with the UTeach program and her M.Ed. in Educational Leadership and Policy Studies from the University of Texas at Arlington. She taught elementary and middle school mathematics for four years. She also served at the Texas Education Agency for three years in a variety of roles, including the Assistant Director of Mathematics and Mathematics Curriculum Specialist in the Curriculum Division and Mathematics Assessment Specialist in the Student Assessment Division. She is currently pursuing her doctoral degree at Southern Methodist University.

**Item Writer 7.** Item Writer 7 earned a Bachelor of Arts degree in Biochemistry and Mathematics from Austin College. She later earned a Master of Science degree in Biochemistry from Baylor College of Medicine. She has taught high school algebra and geometry and has tutored middle school, high school, and college level math. She has a M.Ed. degree from Southern Methodist University and is currently pursuing her doctoral degree at the same university.

## Item Writing Training

All item writers were trained to write items that aligned with the content expectations and item specifications. Training included review of the Item Writing Training Manual and participation in a training conference call with the researchers and project staff. The Item Writing Training Manual provides a detailed description of the principles of universal design for assessment and logistical information about formatting, reviewing, and submitting items. Reviewers received guidelines for writing selected response items, written by recognized experts in item design, and information on the elements of high quality test design. Moreover, reviewers were given sample items illustrating important components of effective items. A glossary of useful terms and a list of relevant websites were provided.

A training meeting was conducted with the item writers to review the content standards and levels of cognitive complexity of the items for Grade 3. Project staff first provided a detailed description of the content by reviewing each CFP for the grade level. Item writers were then provided with the blueprint for the Grade 3 Universal Screener, which delineated the number of items to be written for each CFP and the number of associated cognitive complexity levels to be addressed in item development. Item writers were further assigned an additional CFP on which to design items. Example items for each CFP and respective levels of cognitive complexity were disseminated and discussed. Finally, any additional material in the Item Writing Training Manual was reviewed and discussed until the item writers were confident they understood the content and objectives of the project.

### Item Writing Process

After completing the training and attending the project meeting, item writers were given the item writing template to create items. Upon completion of the items, reviewers submitted items to researchers and project staff for review. At least two internal reviewers provided feedback for each item. Reviewers evaluated items for (1) mathematical accuracy, (2) alignment with the content standards, (3) age-appropriateness of language and graphics for students in Grade 3, and (4) compliance with universal design principles. Reviewer comments were returned to the item writers to revise and resubmit for approval. All finalized items were cross-referenced to the test blueprint and specifically to the content standard to ensure that each standard had a corresponding item. When standards were found without items, items were written.

Once items were accepted, item level information was entered into an Item Database. The Istation graphic design team created all graphics. The finalized items were copy-edited and reviewed by SMU researchers and Istation staff.

## Content-Related Evidence for Validity

Mathematicians and mathematics teachers evaluated all items for accuracy and appropriateness of the content written for the formative assessment item bank for students in Grade 3.

### Mathematician Review

Two mathematicians reviewed all items in Grade 3. Both were professors of mathematics at universities in Texas and held undergraduate and graduate degrees in mathematics. They have 14 and 19 years, respectively, of teaching and researching in mathematics. Both reviewers were female.

The mathematicians were asked to review each item and evaluate the accuracy of the content, precision of the vocabulary, and effectiveness of distractors. The criteria used for item evaluation are as follows:

- Mathematical accuracy of content: Each item was written to reflect an integration of knowledge and skills identified by the NCTM Curriculum Focal Points. Is the item mathematically accurate?
- Precision of mathematical vocabulary: Is the mathematical vocabulary used accurately? Is the mathematical vocabulary precise?
- Appropriateness of the distractors: Most students use an eliminating process to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions from choosing a correct answer in the elimination process. Are the distractors appropriate for the item? Are the distractors mathematically plausible misconceptions?

Items and distractors were evaluated on a 4-point scale for each criterion. A rating of 1 indicated that the item was not accurate, precise, or appropriate; a rating of 2 indicated that the item was somewhat accurate, precise, or appropriate; a rating of 3 indicated that the item was mostly accurate, precise, or appropriate; and a rating of 4 indicated the item was extremely accurate, precise, or appropriate. In instances where the reviewer assigned a score of 1 or 2 for any criterion, recommendations were solicited that would aid in revision.

Overall, the mathematicians rated the items as always mathematically accurate, and the vocabulary and distractors as mostly or always precise and appropriate, respectively. The mathematicians recommended revisions for 34 items. Both reviewers generally noted the following issues on items: wording of the stem or distractors to improve communication of mathematical concepts, the possibility of multiple plausible answer choices, some distractors could be changed to improve the question, unclear graphics, and item difficulty was too high.

We revised most items in response to the recommendations. Items that the reviewers perceived as too difficult were referencing the state content standards when making this determination instead of the CFP. For each of the items in which a reviewer expressed concern, alignment with the CFP was verified. In instances where the mathematician did not provide a suitable suggestion for a revised distractor, we modified the item.

## Teacher Review

Three teachers with experience teaching Grade 3 mathematics reviewed the items. One reviewer was a Caucasian female with over 19 years of teaching early grade school. Another reviewer was an African-American female who had taught early grade school for 18 years and now teaches at the college level. The final reviewer was a Caucasian female with seven years of experience teaching kindergarten, 3rd, and 4th grade. All reviewers were certified to teach middle school mathematics by the state of Texas.

Teachers analyzed each item for appropriate grade-level language and vocabulary, content or concepts, graphics, potential bias in language and/or content, clarity of directions and answers, and effectiveness of distractors. The criteria presented for item evaluation are as follows:

- Appropriateness of language: Is the language used in the item appropriate for students in your grade level? Are the question and response options written so that students in your grade level can understand the meaning of the problem?

- Appropriateness of mathematical vocabulary: Is the mathematical vocabulary representative of pre-requisite or instructional expectations in your grade level?

- Appropriateness of content or concepts: Is the task representative of pre-requisite or instructional expectations in your grade level?

- Appropriateness of visual representation: Is the visual representation (i.e., graphic, table, image) used in the item appropriate for students in your grade level? Can students in your grade level understand the meaning of the visual representation? Is the visual representation of the item clear?

- Bias in language or content: Does the item require background knowledge unrelated to the concept being tested that would differ for students with different backgrounds? Is the language sensitive to students from diverse backgrounds, students with limited English proficiency and students with special needs? Example: "*What is the most appropriate measurement unit for the length of a sub or hoagie?*" may be unfair for students in certain geographic regions and students with diverse background who are unfamiliar with the terms "sub or hoagie".

- Effectiveness of the distractors: Some students use an eliminating process to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions choosing a correct answer in the elimination process. Are the distractors appropriate for the item? Do the distractors discriminate between students with specific misconceptions?

The items and distractors were rated on a scale of 1 to 4 for each criterion. A rating of 1 indicated that the item/distractors were not at all appropriate based on the criterion (or very biased); a rating of 2 indicated that the item/distractors were somewhat appropriate based on the criterion (or somewhat biased); rating of 3 indicated that the item/distractors were appropriate based on the criterion (or not biased); and a rating of 4 indicated that the item/distractors were extremely appropriate based on the criterion (or not biased *and* has multi-cultural components to it). In instances where the teachers provided a rating of 2 or lower, they were asked to provide additional suggestions and comments to improve the item.

Overall, the teachers rated the items as mostly to mostly and always appropriate in regards to language, vocabulary content, visual representation, bias, and effectiveness of distractors. The teachers recommended revising 119 items. For over 30 items, the teachers noted the items as having confusing language. Teachers recommended clarifying the language of some items to make the concept clear for Grade 3 students. For each of these items, the item was either reworded or otherwise modified. The teachers felt that 6 items were too difficult or too easy for Grade 3. These reviewers were referencing the state content standards when making this determination, instead of the CFP. For each of the items in which a reviewer expressed concern, alignment with the CFP was verified.

The research team reviewed all suggestions and made revisions based on teacher feedback. The graphics for two items were noted as being confusing or difficult to read. We created new graphics for these items. The distractors on one item were thought to be too obviously incorrect. For this item, we changed the distractors to be more plausible. Finally, symbols did not display correctly for equations in one item. The technical error was resolved, allowing the graphics to display correctly.

## Conclusions

The purpose of this technical report was to describe the development of the formative assessment item bank. We described the construct underlying the items in reference to the content standards and levels of cognitive complexity. In addition, we described the process for sampling the content assessed in the item bank. Next, we described the item writing procedures and provided the qualifications for the item writers. Finally, we documented the process and outcomes of an external item review by mathematicians and mathematics teachers to document content-related evidence for validity.

## References

American Educational Research Association (AERA), American Psychological Association

(APA), & National Council on Measurement in Education (NCME). (1999). *Standards for educational and psychological testing*. Washington, DC: American Psychological Association.

Ketterlin-Geller, L.R. (2005). Knowing what all students know: Procedures for developing universally designed assessments. *Journal of Technology, Learning, and Assessment, 4(2)*. Available from http://escholarship.bc.edu/jtla/vol4/2

Ketterlin-Geller, L.R. (2008). Testing students with special needs: A model for understanding the interaction between assessment and student characteristics in a universally designed environment. *Educational Measurement: Issues and Practice, 27(3),* 3-16. DOI: 10.1111/j.1745-3992.2008.00124.x

Kilpatrick, J., Swafford, J. & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

## Figure 1

*Content Sampling Matrix*

| | Procedural fluency | | | Conceptual understanding | | | Strategic competence | | | Adaptive reasoning | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CFP | Easy | Medium | Difficult | Easy | Medium | Difficult | Easy | Medium | Difficult | Easy | Medium | Difficult |
| 1 | 10 | 10 | 10 | 10 | 10 | 10 | 7 | 6 | 7 | 7 | 6 | 7 |
| 2 | 10 | 10 | 10 | 10 | 10 | 10 | 7 | 6 | 7 | 7 | 6 | 7 |
| 3 | 10 | 10 | 10 | 10 | 10 | 10 | 7 | 6 | 7 | 7 | 6 | 7 |
| 4 | 10 | 10 | 10 | 10 | 10 | 10 | 7 | 6 | 7 | 7 | 6 | 7 |
| Total By Difficulty | 40 | 40 | 40 | 40 | 40 | 40 | 28 | 24 | 28 | 28 | 24 | 28 |

# Appendix A - State Content Standards Referent Sources

**National Council of Teachers of Mathematics (NCTM) Curricular Focal Points**

The National Council of Teachers of Mathematics (NCTM) Curricular Focal Points were retrieved from http://www.nctmmedia.org/cfp/front_matter.pdf on 4/20/2010. Additional information was also retrieved on 4/20/2010 from: www.nctm.org/focalpoints . The coding system for the NCTM Critical Focal Points can be found under Part II.

**Florida**

Florida's Next Generation Sunshine State Math Standards (adopted 2007) were retrieved on 4/20/2010 from http://www.floridastandards.org/Standards/FLStandardSearch.aspx. Verification of accuracy and currency of the standards was obtained on 5/5/2010 from Florida Department of Education. Big Ideas for each of the grade levels were also verified.

**California**

California's Math Content Standards (adopted 1997) were retrieved on 4/24/2010 from http://www.cde.ca.gov/be/st/ss/documents/mathstandard.pdf . California Green Dot Standards are the selected standards (as of 2006) that appear 85% of the time on California state tests. These green dot standards were retrieved on 4/24/2010 from http://caworldclassmath.com/high_ca_standards.html and etc.usf.edu/flstandards/math/california.ppt . Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the California State Board of Education.

**New York**

The New York State Standards (revised on March 15, 2005) were retrieved on 4/21/2010 from: http://www.bootstrapworld.org/standards/ny/NYMathematicsCoreCurriculum.pdf .Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the New York State Board of Education.

**Texas**

The Texas State Standards for Math (Version 2.1; revised 2010) were retrieved on 4/21/2010 from: http://ritter.tea.state.tx.us/rules/tac/chapter111/index.html. Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the Texas State Board of Education. The Texas Education Agency (TEA) released a 2010 document entitled *Texas Response to Curriculum Focal Points: Kindergarten through Grade 8 Mathematics* that included coordinating TEKS.

**Common Core Standards**

The Common Core Standards in Mathematics were retrieved on June 10, 2011 from http://www.corestandards.org/the-standards/mathematics . These standards were published in 2010.

They were developed as part of an initiative led by National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).

**Virginia**

Virginia's Standards for Learning Document for Mathematics (adopted 2009 for full implementation in 2011-12) were retrieved on June 10, 2011 from www.doe.virginia.gov/testing/sol/standards_docs/mathematics/review.shtml . Verification of accuracy and currency of the standards was obtained from Istation on June 10, 2011. The Curriculum Frameworks documents were referenced to determine the essential knowledge and skills students are expected to learn for each grade.

| GRADE 3 MATHEMATICS CURRICULUM FOCAL POINTS | |
|---|---|
| *CFP 1: Number and Operations and Algebra*<br>**Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts** | |
| 3.1A.1 | Students understand the **meanings of multiplication and division of whole numbers through the use of representation**s (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines for multiplication, and successive subtraction, partitioning, and sharing for division). |
| 3.1B.1 | Identify examples of the **identity and commutative properties** for addition and multiplication. |
| 3.1B.2 | Students **use properties of addition and multiplication** [e.g., commutativity, associativity, distributive property, (including identity and zero properties for New York)] to multiply whole numbers. |
| 3.1C.1 | **Develop fluency** with single-digit multiplication facts |
| 3.1C.2 | Demonstrate fluency and **apply** single-digit division facts |
| 3.1C.3 | Students apply increasingly sophisticated strategies based on properties of multiplication and division to **solve multiplication and division problems involving basic facts**. [multiply and divide within 100] |
| 3.1D.1 | Develop strategies for selecting the appropriate computational and operational method in problem solving situations |
| 3.1D.2 | Write number sentences to represent equivalent mathematical relationships (e.g., 4 x 3 = 14 - 2). |

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| 3.1D.3 | Use the inverse relationships between addition/subtraction and multiplication/division to solve related basic fact sentences. For example, 5 + 3 = 8 and 8 – 3 = __; 4 × 3 = 12 and 12 ÷ 4 = __. |
| 3.1D.4 | By comparing a variety of solution strategies, students relate **multiplication and division as inverse operations**. |
| 3.1D.5 | Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8. |
| 3.1E.1 | Recognize repeating and growing numeric and geometric patterns (e.g., skip counting, addition tables, and multiplication tables). |
| 3.1E.2 | Describe and extend numeric (+, -) and geometric patterns |
| 3.1E.3 | Create and analyze patterns and relationships involving multiplication and division. |
| 3.1E.4 | Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. |
| 3.1F.1 | Students build a foundation for later understanding of functional relationships by describing relationships in context with such statements as, “The number of legs is 4 times the number of chairs.” |
| 3.1G.1 | Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7. |
| 3.1H.1 | Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8. |

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| 3.1I.1 | Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \_ \div 3$, $6 \times 6 = ?$ |
| 3.1J.1 | Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. |
| 3.1K.1 | Recognize real world situations in which an estimate (rounding) is more appropriate |
| 3.1K.2 | Determine whether an estimate or an exact answer is an appropriate solution for practical addition and subtraction problems situations involving single- step and multistep problems. |
| 3.1K.3 | Assess the reasonableness of answers (two-step word problems) using mental computation and estimation strategies including rounding. |
| 3.1.L.1 | Use a variety of strategies to add and subtract 3-digit numbers (with and without regrouping) |
| 3.1.L.2 | Students develop their understanding of numbers by building their facility with **mental computation** (addition and subtraction in special cases, such as 2,500 + 6,000 and 9,000 – 5,000). |
| 3.1.M.1 | Students develop their understanding of numbers by building their facility with **computational estimation.** |
| 3.1.N.1 | Students develop their understanding of numbers by building their facility with **paper-and-pencil computations.** |
| 3.1.O.1 | Estimate numbers up to 500 |
| 3.1.O.2 | Represent, compute, estimate, and solve problems using numbers through hundred thousands. |
| 3.1.P.1 | Solve non-routine problems by making a table, chart, or list and searching for patterns. |

| *CFP 2: Number and Operations*<br>**Developing an understanding of fractions and fraction equivalence** | |
|---|---|
| 3.2A.1 | Students develop an **understanding of the meanings and uses of fraction**s to represent parts of a whole, parts of a set, or points or distances on a number line. |
| 3.2A.2 | Understand and recognize the **meaning of numerator and denominator** in the symbolic form of a fraction |
| 3.2A.3 | Name and write fractions (including mixed numbers) represented by a **model** to include halves, thirds, fourths, eighths, tenths, and twelfths. |
| 3.2A.4 | Name and write fractions and mixed numbers represented by **drawings** or **concrete materials**. |
| 3.2B.1 | Students understand that the **size of a fractional part is relative to the size of the whole,** and they use fractions to represent numbers that are equal to, less than, or greater than 1. |
| 3.2C.1 | **Compare and order unit fractions** (1/2, 1/3, 1/4) and find their approximate locations on a number line |
| 3.2C.2 | Students solve problems that involve **comparing and ordering fractions** by using models, benchmark fractions, or common numerators or denominators. |
| 3.2D.1 | **Use concrete materials and pictures to mode**l at least halves, thirds, fourths, eighths, tenths, and twelfths. |
| 3.2D.2 | **Represent a given fraction or mixed number, using concrete materials, pictures, and symbols.** For example, write the symbol for one-fourth and represent it with concrete materials and/or pictures. |
| 3.2D.3 | **Add and subtract with proper fractions** having like denominators of 12 or less, using concrete materials and pictorial models representing area/regions (circles, squares, and rectangles), length/measurements (fraction bars and strips), and sets (counters). |

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| 3.2D.4 | Students understand and use models, including the number line, to **identify equivalent fractions.** |
| 3.2E.1 | Understand the place value structure of the base ten number system: 10 ones = 1 ten 10 tens = 1 hundred 10 hundreds = 1 thousand |
| 3.2E.2 | Building on their work in grade 2, students **extend their understanding of place value** to numbers up to 10,000 in various contexts. |
| 3.2E.3 | Use place value understanding to **round whole number**s to the nearest 10 or 100. |
| 3.2E.4 | Round a given whole number, 9,999 or less, to the nearest ten, hundred, and thousand. |
| 3.2E.5 | Solve problems, using rounding of numbers, 9,999 or less, to the nearest ten, hundred, and thousand. |
| 3.2F.1 | Students apply their understanding of place value to the task of representing numbers in different equivalent forms (e.g., expanded notation). |
| 3.2G.1 | Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. |
| 3.2G.2 | 3.3 (A) The student is expected to model addition and subtraction using pictures, words, and numbers. |
| 3.2G.3 | 3.3 (B) The student is expected to select addition or subtraction and use the operation to solve problems involving whole numbers through 999. |
| 3.2G.4 | Add or subtract two whole numbers, each 9,999 or less. |
| 3.2H.1 | Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. |

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| 3.2I.1 | Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. |
| 3.2J.1 | Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. |
| 3.2K.1 | Use the **symbols <, >, =** (with and without the use of a number line) to compare whole numbers and unit fractions (1/2, 1/3, 1/4, 1/5, 1/6, and 1/10) |
| 3.2K.2 | Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. |
| 3.2L.1 | Know and understand that 25 cents is a 1/4 of a dollar, 50 cents is 1/2 of a dollar, and 75 cents is 3/4 of a dollar. |
| 3.2.M.1 | Skip count by 25's, 50's, 100's to 1,000 |
| 3.2.N.1 | Read and write whole numbers to 1,000 |
| 3.2.N.2 | Read six-digit numerals orally. |
| 3.2.N.3 | Write six-digit numerals that are stated verbally or written in words. |
| 3.2.O.1 | Compare and order numbers to 1,000 |
| 3.2.P.1 | Use a variety of strategies to compose and decompose three-digit numbers |
| 3.2.Q.1 | Identify odd and even numbers |
| 3.2.R.1 | Develop an understanding of the properties of odd/even numbers as a result of addition or subtraction |

*CFP 3: Geometry*
**Describing and analyzing properties of two-dimensional shapes**

***Data Analysis Connection to the Focal Point*** **includes students using addition, subtraction, multiplication and division of whole numbers come into play as students construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to solve problems.**

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| 3.3A.1 | **Identify models and pictures of plane geometric figures** (circle, square, rectangle, and triangle) **and solid geometric figures** (cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by name. |
| 3.3A.2 | Students describe, analyze, compare, and classify **two-dimensional shapes by their sides and angles**. |
| 3.3B.1 | Students **connect attributes of two dimensional shapes** to their definitions. |
| 3.3C.1 | Students investigate, describe, and reason about **decomposing, combining, and transforming polygons** to make other polygons. |
| 3.3D.1 | Through building, drawing, and analyzing two-dimensional shapes, students **understand attributes and properties of two-dimensional space.** |
| 3.3E.1 | Identify **congruent and similar figures** |
| 3.3E.2 | The student is expected to identify **congruent two-dimensional figures**. |
| 3.3E.3 | Students use **attributes and properties** of two dimensional shapes in solving problems, including applications **involving congruence.** |
| 3.3F.1 | Identify examples of points, line segments, rays, angles, and lines. |
| 3.3F.2 | Draw representations of points, line segments, rays, angles, and lines, using a ruler or straightedge. |

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| 3.3F.3 | Identify and construct lines of symmetry |
| 3.3F.4 | Students use **attributes and properties** of two dimensional shapes in solving problems, including applications **involving symmetry.** |
| 3.3G.1 | Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape. |
| 3.3H.1 | Name, describe, compare, and sort three-dimensional shapes: cube, cylinder, sphere, prism, and cone |
| 3.3I.1 | Identify the faces on a three-dimensional shape as two-dimensional shapes |
| 3.3J.1 | Compare and contrast characteristics of plane and solid geometric figures (e.g., circle/sphere, square/cube, triangle/square pyramid, and rectangle/rectangular prism), by counting the number of sides, angles, vertices, edges, and the number and shape of faces. |
| 3.4A.1 | Use addition, subtraction, multiplication, and division of whole numbers to **construct** frequency tables, bar graphs, picture graphs, and line plots |
| 3.4B.1 | Use addition, subtraction, multiplication, and division of whole numbers to analyze frequency tables, bar graphs, picture graphs, and line plots |
| 3.4C.1 | Use addition, subtraction, multiplication, and division of whole numbers to use frequency tables, bar graphs, picture graphs, and line plots to solve problems. |
| 3.4K.1 | Formulate questions about themselves and their surroundings |
| 3.4K.2 | Design data investigations to answer formulated questions, limiting the number of categories for data collection to four. |

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| 3.4K.3 | Collect data using observation and surveys, and record appropriately |
| 3.4K.4 | Identify the parts of pictographs and bar graphs |
| 3.4K.5 | Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets. |
| 3.4K.6 | Select a correct interpretation of a graph from a set of interpretations of the graph, where one is correct and the remaining are incorrect. For example, a bar graph containing data on four ways to cook or prepare eggs — eaten by students show that more students prefer scrambled eggs. A correct answer response, if given, would be that more students prefer scrambled eggs than any other way to cook or prepare eggs. |
| 3.4K.7 | Analyze and interpret information from picture and bar graphs, with up to 30 data points and up to 8 categories, by writing at least one sentence. |
| 3.4K.8 | Describe the categories of data and the data as a whole (e.g., data were collected on four ways to cook or prepare eggs — scrambled, fried, hard boiled, and egg salad — eaten by students). |
| 3.4K.9 | State the relationships between pictographs and bar graphs |
| 3.4K.10 | Formulate conclusions and make predictions from graphs |
| 3.4K.11 | Define probability as the chance that an event will happen. |
| 3.4K.12 | List all possible outcomes for a given situation (e.g., heads and tails are the two possible outcomes of flipping a coin). |
| 3.4K.13 | Identify the degree of likelihood of an outcome occurring using terms such as impossible, unlikely, as likely as, equally likely, likely, and certain. |

*Measurement Connections to Focal Points*
**Students in grade 3 strengthen their understanding of fractions as they confront problems in linear measurement that call for more precision than the whole unit allowed them in their work in grade 2. They develop their facility in measuring with fractional parts of linear units. Students develop measurement concepts and skills through experiences in analyzing attributes and properties of two-dimensional objects. They form an understanding of perimeter as a measurable attribute and select appropriate units, strategies, and tools to solve problems involving perimeter.**

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| 3.4D.1 | Students **measure with fractional parts of linear units**. |
| 3.4E.1 | Students develop measurement concepts and skills through experiences in **analyzing attributes and properties of two-dimensional objects.** |
| 3.4F.1 | Students **understand perimeter** as a measurable attribute |
| 3.4G.1 | Students select appropriate **units, strategies, and tools to solve problems involving perimeter**. |
| 3.4G.2 | Measure each side of a variety of polygons and add the measures of the sides to determine the perimeter of each polygon. |
| 3.4H.1 | Relate unit fractions to the face of the clock: Whole = 60 minutes, 1/2 = 30 minutes, 1/4 = 15 minutes |
| 3.4H.2 | Identify the number of minutes in an hour and the number of hours in a day. |
| 3.4H.3 | Identify equivalent relationships observed in a calendar, including the number of days in a given month, the number of days in a week, the number of days in a year, and the number of months in a year. |
| 3.4H.4 | Tell and write time to the nearest minute (digital and analog) and measure time intervals (and elapsed time) in minutes. Solve word problems involving addition and subtraction of time intervals (and elapsed time) in minutes, e.g., by representing the problem on a number line diagram. |

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| 3.4I.1 | Select and use standard (customary) and non-standard units to estimate measurements |
| 3.4I.2 | Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l) [and English units for volume and weight]. |
| 3.4J.1 | Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. |
| 3.4L.1 | Recognize area as an attribute of plane figures and understand concepts of area measurement... A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. |
| 3.4M.1 | Recognize area as an attribute of plane figures and understand concepts of area measurement… A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. |
| 3.4N.1 | Estimate and use U.S. Customary and metric units to measure area and perimeter. |
| 3.4N.2 | Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). |
| 3.4O.1 | Relate area to the operations of multiplication and addition… Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. |
| 3.4P.1 | Relate area to the operations of multiplication and addition... Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. |
| 3.4Q.1 | Relate area to the operations of multiplication and addition... Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. |

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| 3.4R.1 | Relate area to the operations of multiplication and addition... Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. |
| 3.4S.1 | Recognize capacity as an attribute that can be measured |
| 3.4T.1 | Compare capacities (e.g., Which contains more? Which contains less?) |
| 3.4U.1 | Measure capacity, using cups, pints, quarts, and gallons |
| 3.4V.1 | Count and represent combined coins and dollars, using currency symbols ($0.00) |
| 3.4.W.1 | Compare the values of two sets of coins or bills, up to $5.00, using the terms greater than, less than, and equal to. |
| 3.4.X.1 | Make change from $5.00 or less. |
| 3.4.Y.1 | Read temperature to the nearest degree from real Celsius and Fahrenheit thermometers and from physical models (including pictorial representations) of such thermometers. |