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Exploring Research in Algebra:

Algebra Readiness in the Elementary Classroom

February 15th, 2013

Research in Mathematics Education

Math Slide (Figure 1)

The share of American males studying math-intensive subjects has fallen; it declined dramatically when the share completing college rose, but recently has slid downward along with college completion rates.

SOURCE: American Community Survey of 2009 and 2010

Research on Algebraic Understanding

- Algebra has often been characterized as developmentally constrained due to its inherent abstractness (e.g., Kieran, 1981, 1985; Vergnaud, 1985)

- Research in the former Soviet Union suggested that young children could generalize arithmetic, moving from particular to generalized numbers, learning to use variables and covariation in word problems, and focusing on the concept of *function (*Davydov, 1991, Bodanskii, 1991)

Research on Algebraic Understanding

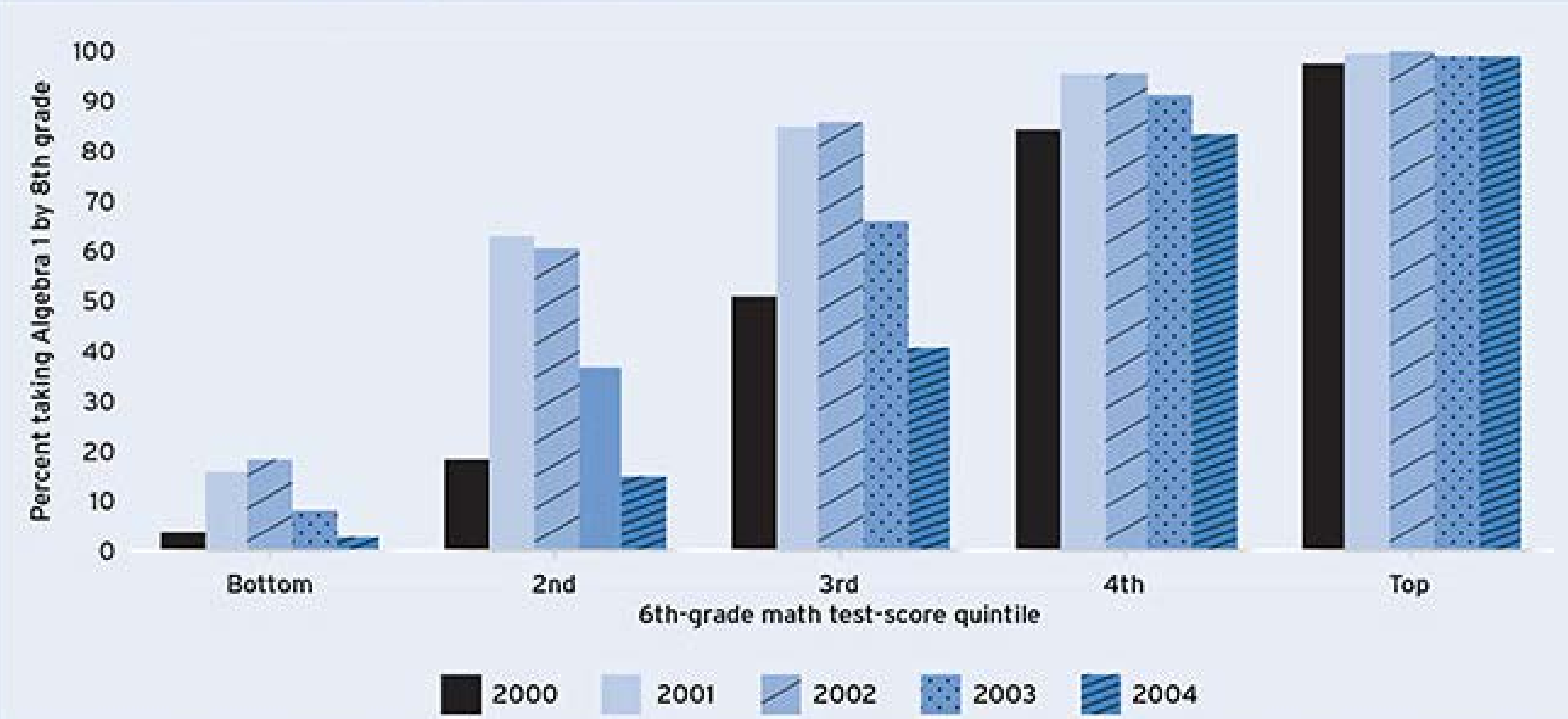
- Recent research suggests that inappropriate instruction may have had a decisive role in the poor results from early studies of algebraic reasoning among adolescents (Booth, 1988; Schliemann & Carraher, 2002).

- Studies of systemic algebra instruction have provided equivocal findings (Clotfelter, Ladd, &Vigdor, 2012; Cortes, Goodman, & Nomi, 2013)

Effects of Accelerating Algebra

Course Reversal (Figure 3)

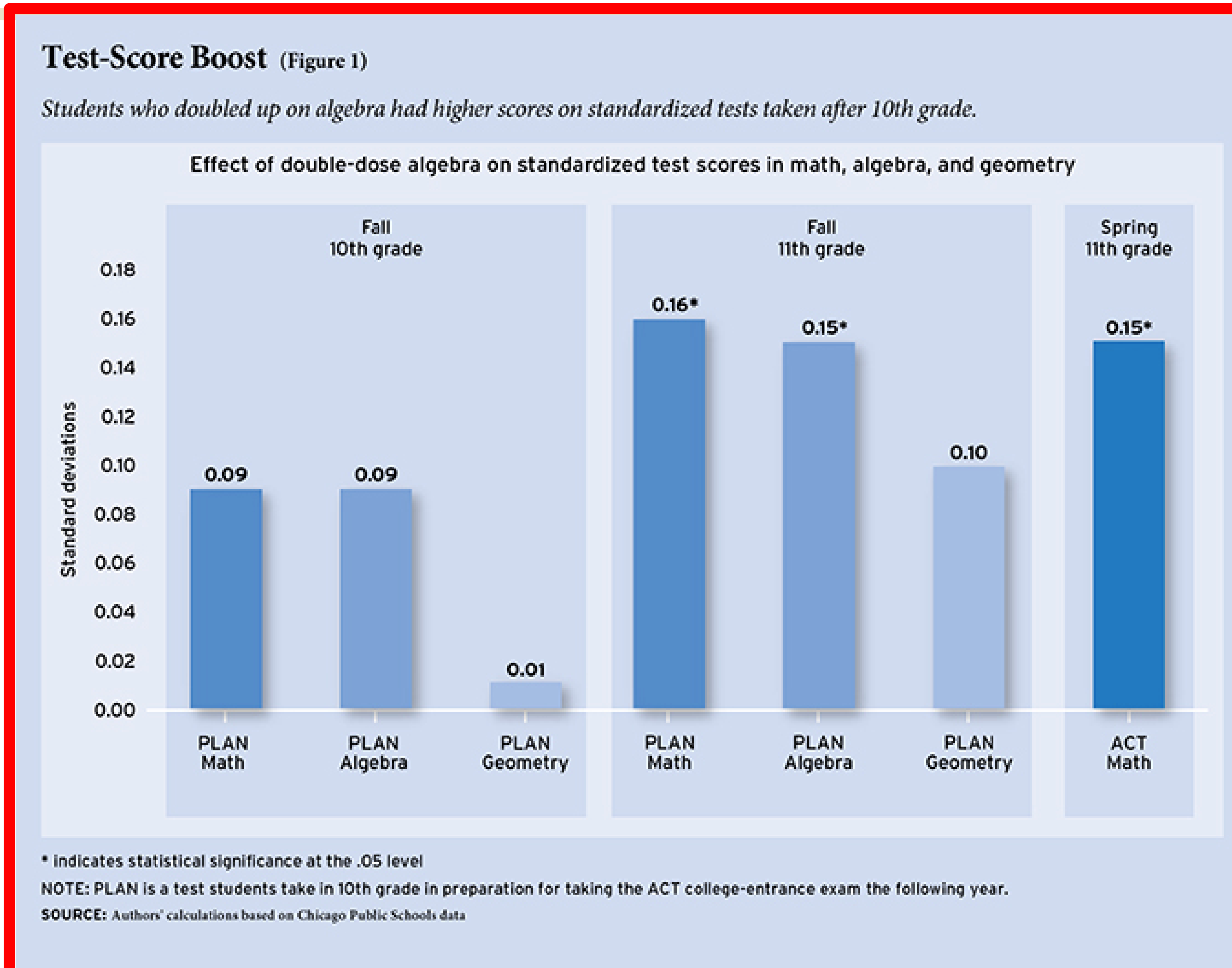
In 2001 and 2002, Charlotte-Mecklenburg's algebra acceleration policy expanded access to Algebra I by 8th grade for less-skilled students, but the change was short-lived.



Note: Figure shows the share of Charlotte-Mecklenburg students taking Algebra I by 8th grade, by 6th-grade math test-score quintile and year entering 7th grade.

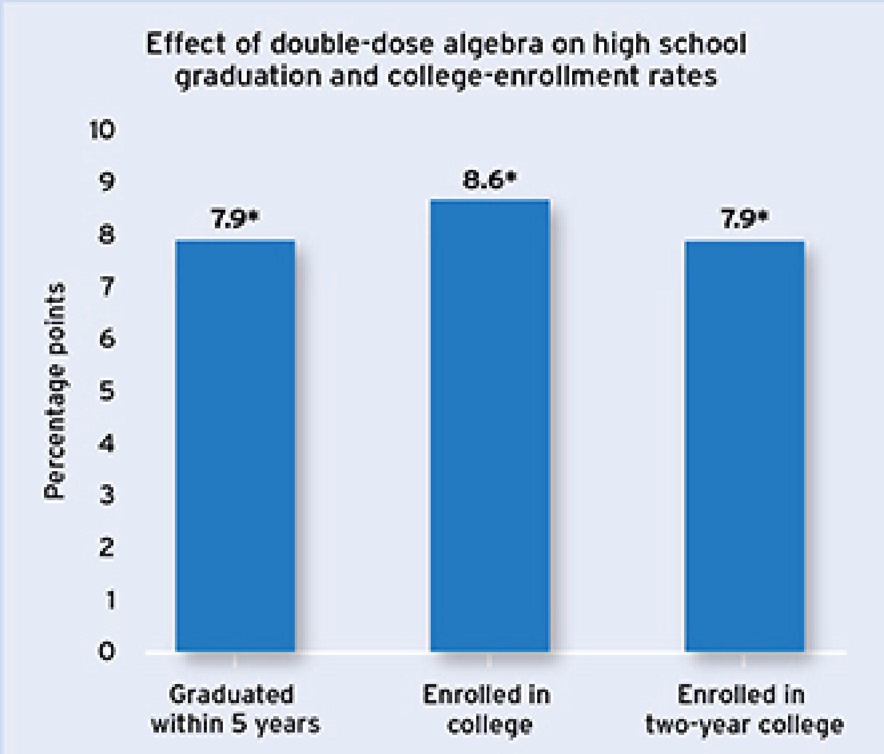
SOURCE: Charles T. Clotfelter, Helen F. Ladd, and Jacob L. Vigdor, 2012. "The Aftermath of Accelerating Algebra: Evidence from a District Policy Initiative," NBER Working Papers 18161, National Bureau of Economic Research, Inc.

Impact of Double-Doses of Algebra



High Impact (Figure 2)

Double-dose algebra increased the percentage of students who graduated from high school and of those who enrolled in college, with most choosing two-year institutions.

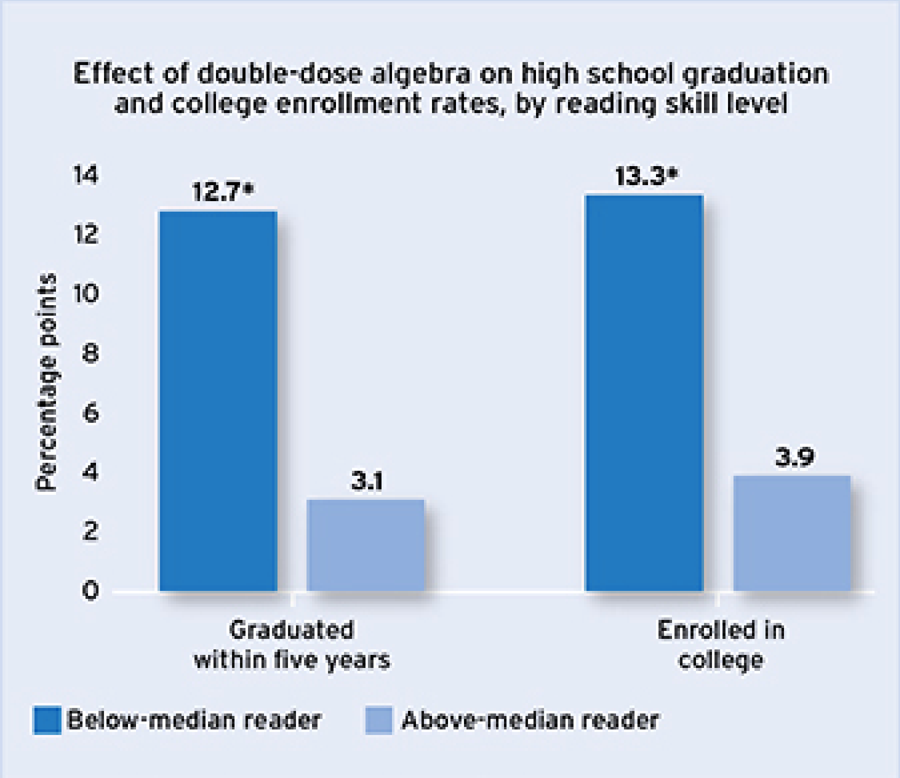


* indicates statistical significance at the .05 level

SOURCE: Authors' calculations based on Chicago Public Schools data and National Student Clearinghouse data

Reading and Writing in Algebra (Figure 3)

Students with weak reading skills benefited more from the algebra support class than otherwise similar students, perhaps because reading and writing were central to the instructional model.



* indicates statistical significance at the .05 level

SOURCE: Authors' calculations based on Chicago Public Schools data and National Student Clearinghouse data

Critical Topics for Teaching and Learning Algebra

(1) Variables and constants

(2) Decomposing and setting up word problems

(3) Symbolic manipulation

(4) Functions

(5) Inductive reasoning and mathematical induction

Milgram (2005)

“A good teacher walks the edge between the structure of mathematics and the development of a child by considering a progression of strategies, the big ideas involved, and the emergent models.”

Developing an Essential Understanding of Algebraic Thinking

- Arithmetic as a Context for Algebraic Thinking
  - 5 Essential Understandings
- Equations
  - 3 Essential Understandings
- Variables
  - 5 Essential Understandings
- Quantitative Reasoning
  - 2 Essential Understandings
- Functional Thinking
  - 6 Essential Understandings

Blanton, Levi, Crites, Dougherty, 2011

Arithmetic as a context for algebraic thinking

- The Fundamental Properties of number and operations govern how operations behave and relate to one another
- The Fundamental Properties are essential to computation
- The Fundamental Properties are used more explicitly in some computation strategies than in others
- Simplifying algebraic expressions entails decomposing quantities in insightful ways
- Generalizations in arithmetic can be derived from the fundamental properties.

Arithmetic as a context for algebraic thinking

- "Historically, arithmetic and algebra were treated as distinct fields of study."
- However, a true understanding of arithmetic also includes reasoning about the fundamental properties.
- Generalizations can be formed through exploration:
  - If you add a number to a given number and then subtract that same number, the given number stays the same.

$$a + b - b = a$$

  - An odd number plus an odd number is an even number

Fundamental Properties

Properties of Addition	• Associative • Commutative • Additive Identity • Additive Inverse
Properties of Multiplication	• Associative • Commutative • Multiplicative Identity • Multiplicative Inverse
Distributive Property of Multiplication over Addition	• Distributive

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Property significance when learning combinations

- The number of addition combinations and multiplication combinations to learn are cut in half when the commutative property is applied.

$$8 + 5 = 5 + 8 \qquad 9 \times 2 = 2 \times 9$$

- When combinations to learn are "chunked" and combined with the commutative and associative properties, students can compute long strings of numbers more efficiently.

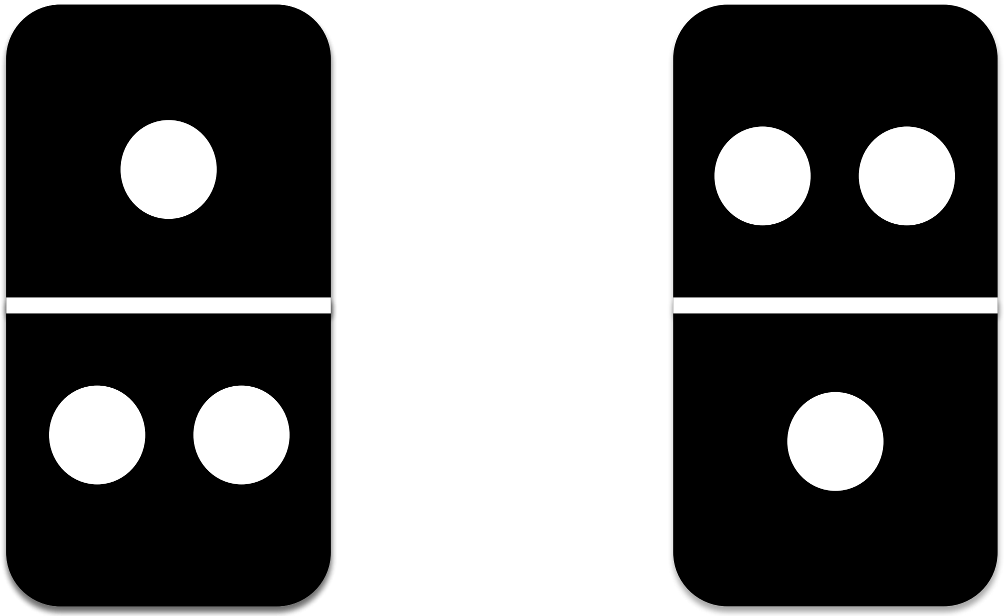
Addition	
Adding 0	+1 or +2
Make 10	Up Over 10
Doubles	Near Doubles

Multiplication	
Zeros	Ones
Doubles	Fives
Nifty Nines	Use the facts you know

Use Mini-Lessons

- 10 minutes a day
- Focus on computational strategies and forming generalizations
- Select problems carefully
- Different types of structures
  - String of problems presented individually but share a relationship
  - Greater than, less than, or equal to
  - True or False
- All answers are valued and explored.

Reasoning with Fundamental Properties



$$1 + 2 = 2 + 1$$

Reasoning with Fundamental Properties

9 + 1

9 + 7 + 1

1 + 6 + 9

Reasoning with Fundamental Properties

$$(4 + 9) + 2 \;\square\; 4 + (9 + 2)$$

$$43 + 17 \;\square\; 17 + 33$$

$$(568 + 153) + 468 \;\square\; 658 + (153 + 468)$$

Reasoning with Fundamental Properties

59 x 16 ☐ 16 x 15

4 x 5 ☐ 5 x 20

(65 x 2) x 1 ☐ 5 x (2 x 1)

13 (15 x 10) ☐ 13 x 130

Equations

- The equals sign is a symbol that represents a relationship of equivalence

$$9 + 5 = 8 + 6$$

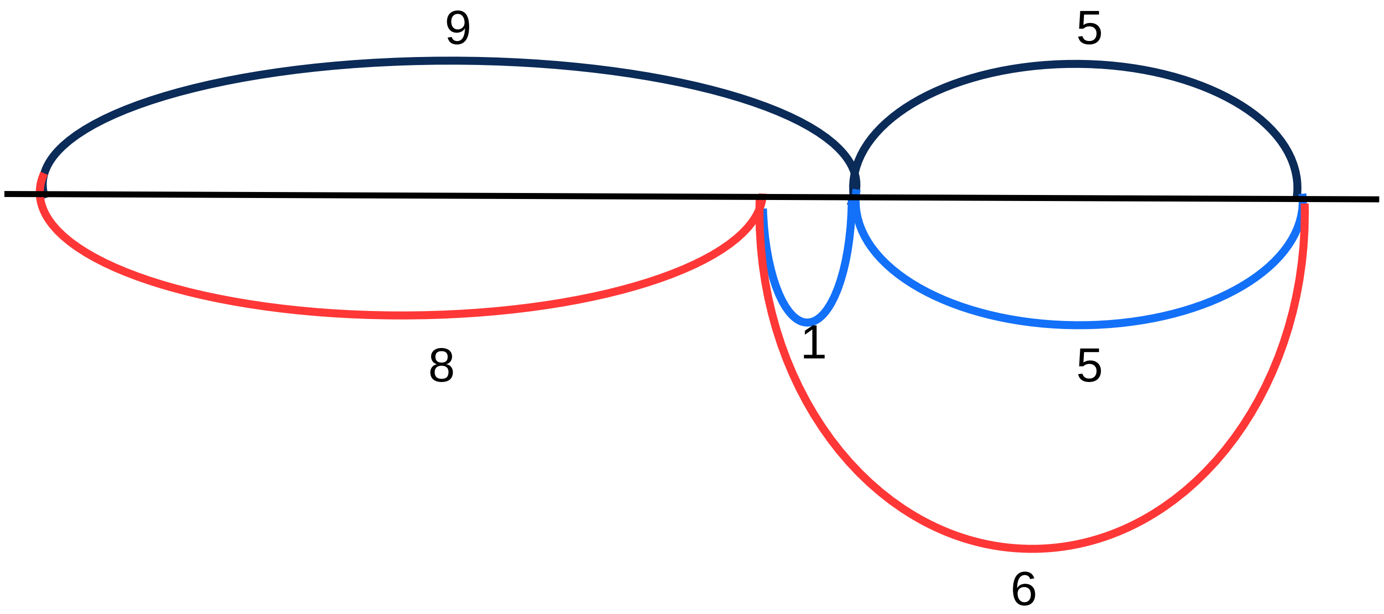
$$13 + 8 + 6 = 5 + 9 + 13$$

$$n + 13 + 9 + 5 = 6 + 8 + 13 + n$$

$$8 + 6 = 5 + 9 + n$$

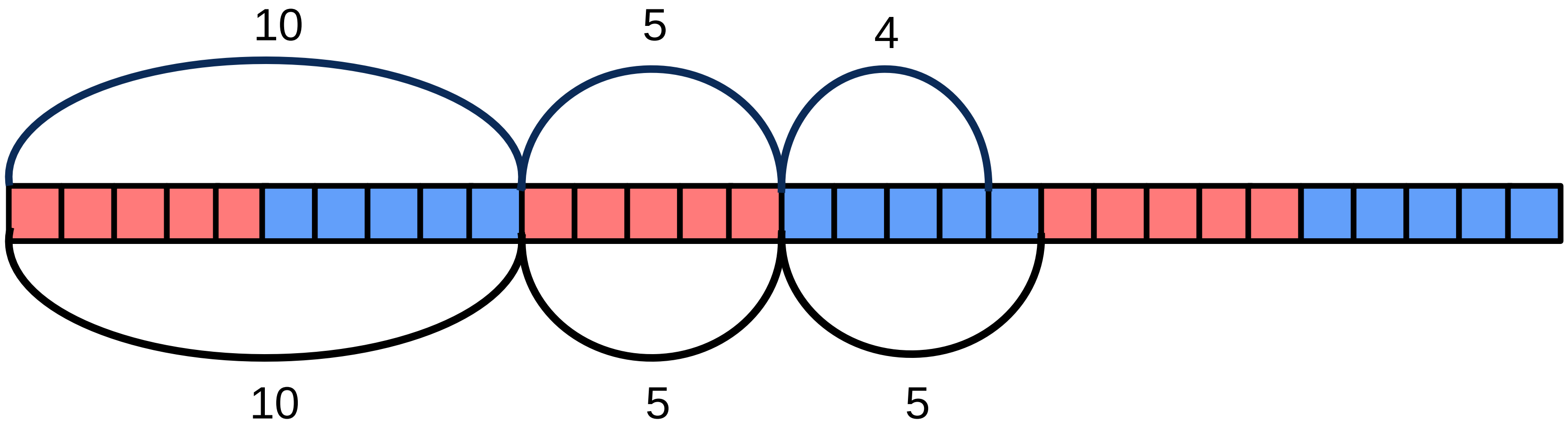
The double number line

8 + 6 = 9 + 5



The double number line

$$5 + 4 + 10 \neq 10 + 5 + 5$$



Equations

- Equations can be reasoned about in their entirety rather than as a series of computations to execute

Column A	Column B
$48 \times 67 \times 6 = k$	$347 \times 25 \times 4 = k$
$346 \times 398 \div 42 = t$	$398 \times 746 \div 746 = d$
$978 + 778 = 394 + y$	$378 + 794 = 778 + j$
$475 \times 2365 = 352 \times w$	$8790 \times 598 = 879 \times n$

Blanton et. al, 2011

Equations

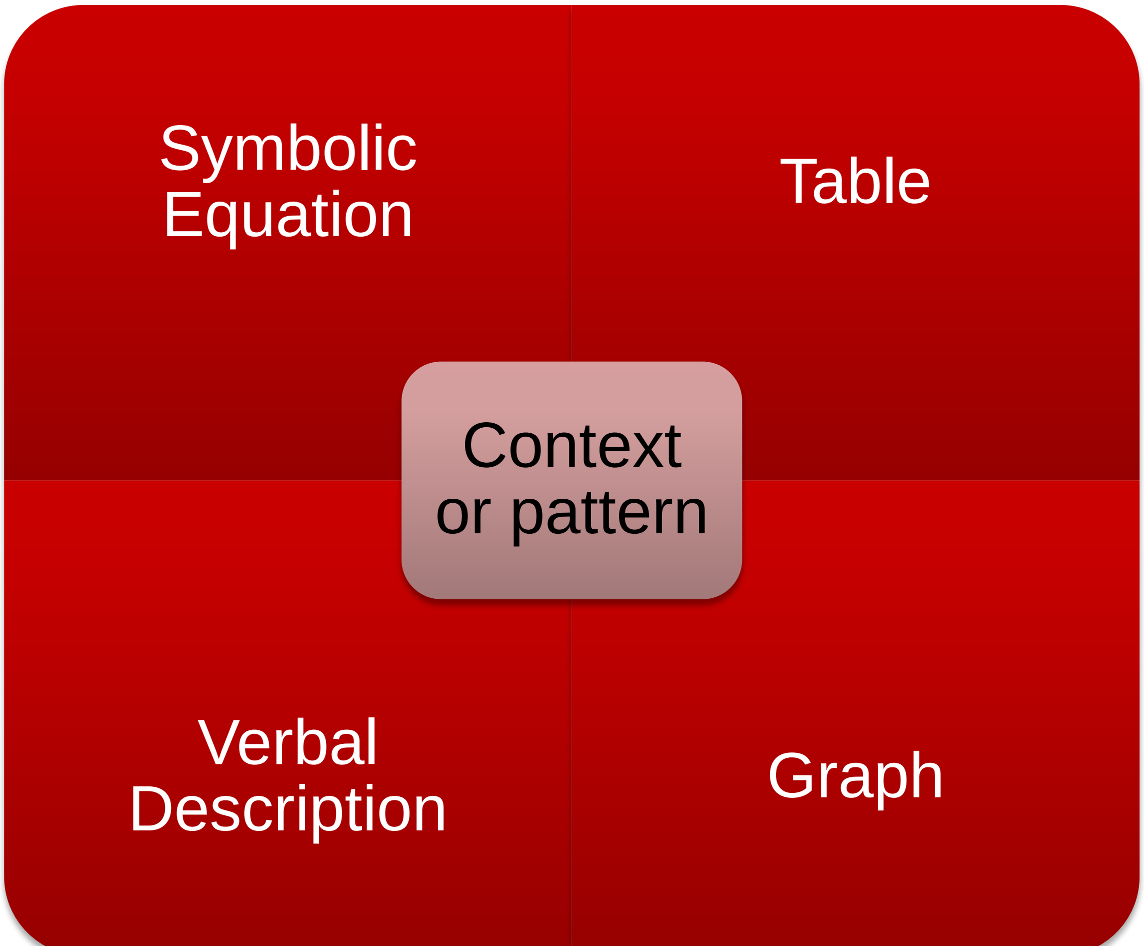
- Equations can be used to represent problem situations
  - The way we solve a problem does not always match the equation that represents the situation in the problem.

JaeQwan is making flowerpots.
One flowerpot takes ¾ of a pound of clay.
How many flowerpots can JaeQwan make with
4½ pounds of clay?

Representation	Ways to solve
$m \times \frac{3}{4} = 4\frac{1}{2}$	$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = 4\frac{1}{2}$ $4\frac{1}{2} \div \frac{3}{4} = 6$

Functional Thinking

- Expressing those relationships in multiple ways



Blanton et. al, 2011

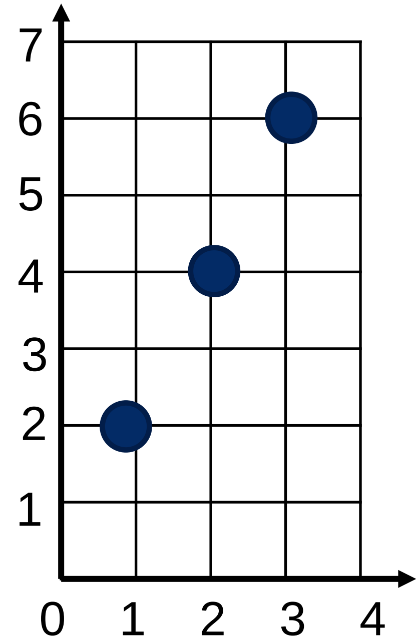
Van de Walle et. al, 2013

Functional Thinking

$y = 2x$

Position	1	2	3
# of circles	2	4	6

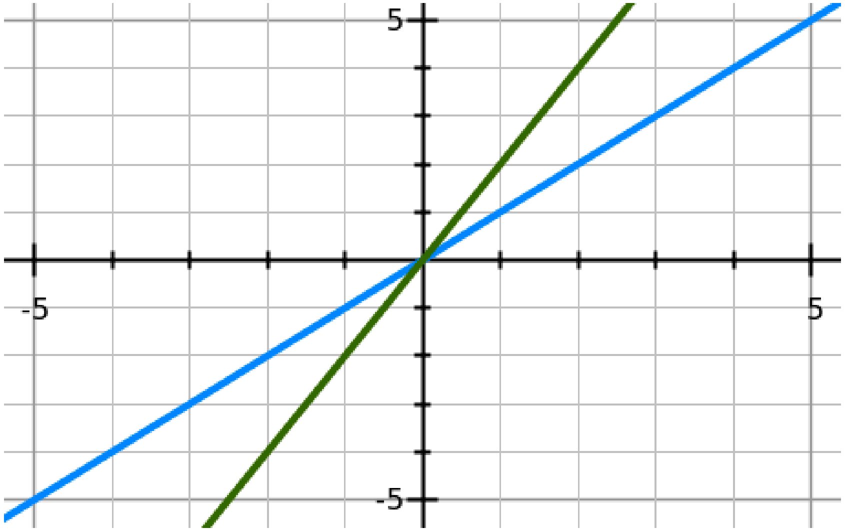
The number of circles is 2 times the position in the pattern.

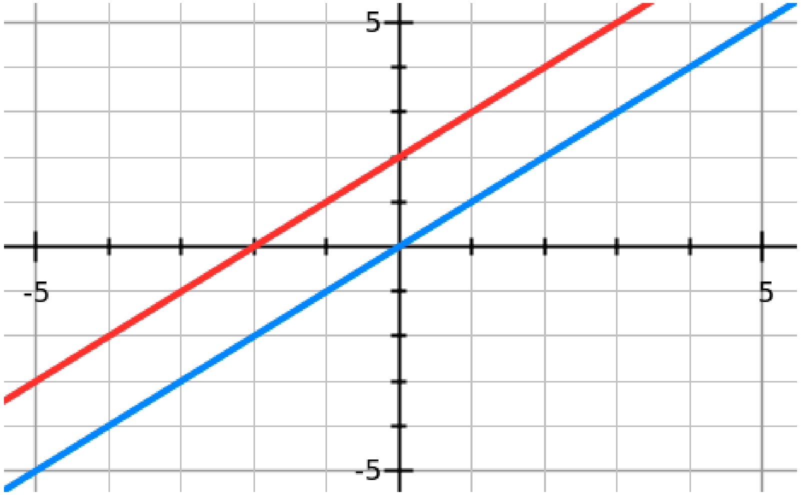


Functions

$y = 2x$

$y = x + 2$

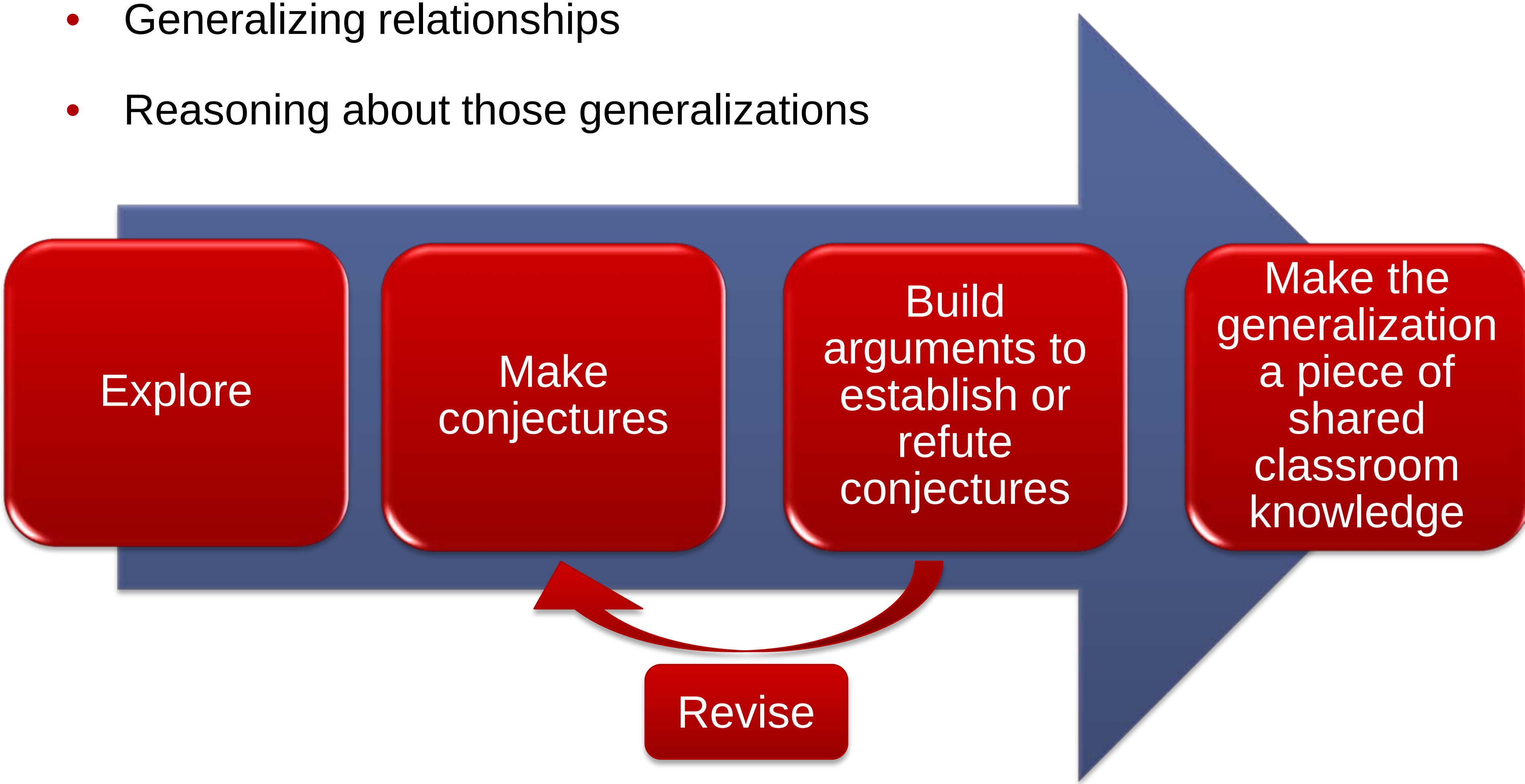




$y = x$

Functional Thinking

- Generalizing relationships
- Reasoning about those generalizations



Blanton et. al, 2011

Kaput et. al, 2008

4 Instructional Goals

- Represent: Provide multiple ways for children to systematically represent algebraic situations.
- Question: Ask questions that encourage children to think algebraically.
- Listen: Listen to build on children’s thinking
- Generalize: Help children develop and justify their own conjectures

Kaput et. al, 2008

Teachers are facilitators

"When I'm working on a problem it's like climbing a mountain. Sometimes I can't even see where I'm going. It is one foot in front of another. And then I reach a point where all of a sudden the vistas open up and I can go down easily for a while, only to eventually reach another climb."

Fosnot and Jacob, 2010

Build the learners capacity to make the climb

categorize

build relations

Develop the mathematician

evaluate

examine

Don't fix the mathematician

cohesive structures

compare

Every action we take should develop the novice mathematicians in front of us

References

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